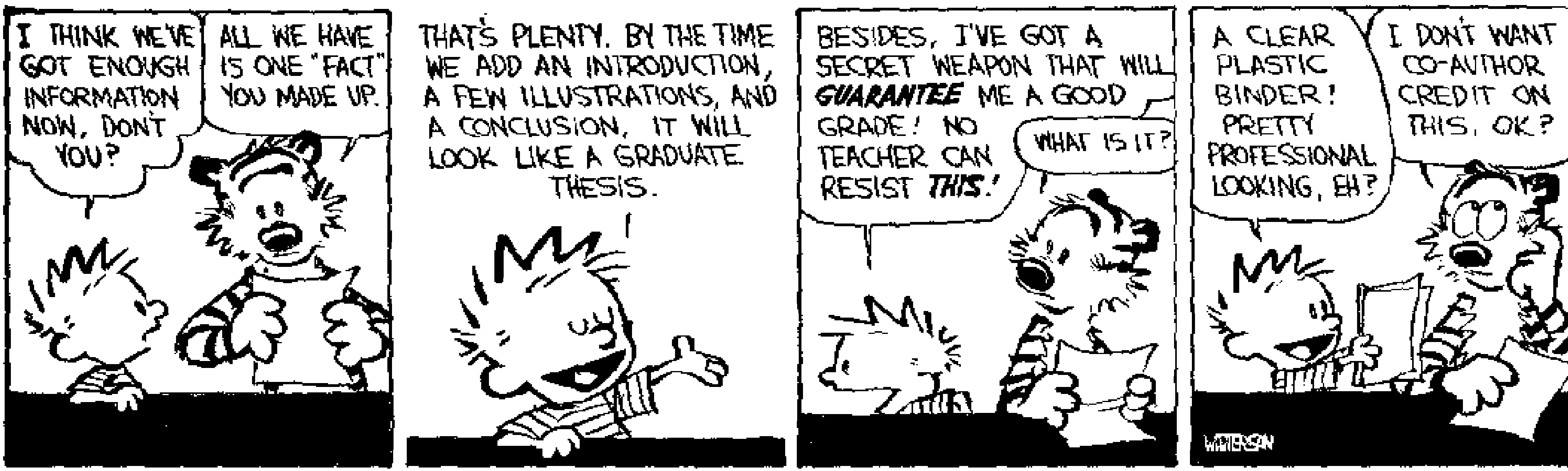
(powerpoint presentation)



The coupling of the lower atmosphere to the thermosphere via gravity wave excitation, propagation and dissipation

Sharon Vadas
NWRA/CoRA division

CEDAR Prize Lecture, June 17, 2008

Big thank you to Dave Fritts for many years of support and encouragement!

Big thank you to NSF Aeronomy
(and program managers Bob Robinson and Bob Kerr)
for my 2002 and 2005 Aeronomy grants----
without these grants, I likely would not have had the resources and freedom to pursue this research.

Papers for which this CEDAR Prize Lecture was awarded:

- Vadas and Fritts, 2004, "Thermospheric responses to gravity waves arising from mesoscale convective complexes", *JASTP*, **66,** 781-804.

- Vadas and Fritts, 2005, " Thermospheric responses to
- gravity waves: Influences of increasing viscosity and thermal diffusivity", *JGR*, **110**, doi:10.1029/2004JD005574.

- Vadas and Fritts, 2006, " Influence of solar variability on gravity wave structure and dissipation in the thermosphere from tropospheric convection", *JGR*, **111,** doi:10.1029/2005JA011510.

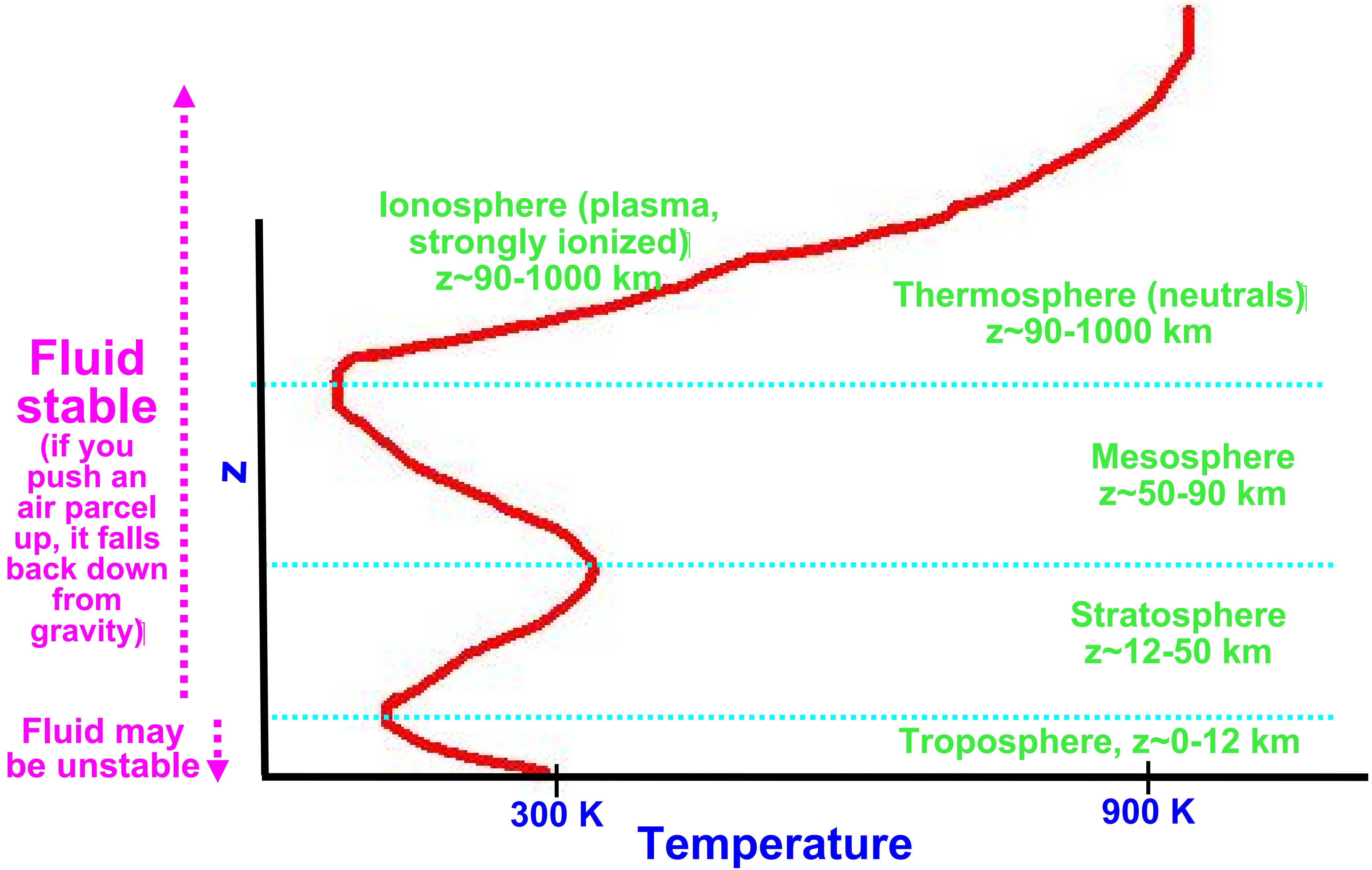
- Vadas, 2007, "Horizontal and vertical propagation and dissipation of gravity waves in the thermosphere from lower atmospheric and thermospheric sources", *JGR*, **112,** doi:10.1029/2006JA011845.

- Vadas and Nicolls, 2008, "Using PFISR measurements and gravity wave dissipative theory to determine the neutral, background thermospheric winds", *GRL*, **35,** doi:10.1029/2007GL031522.

Rothera Base, Adelaide Island, Antartica
(looking towards Antartic Penisula)

Adelaide Island, Antartica

The Earth's Atmosphere:



In the stable part of the atmosphere, there are **only 2 linear responses** to a "small" disturbance:

(e.g., wind flow over mountains, convective overshoot)

- **Sound Waves** - generally not important energetically since typical disturbance velocities are much slower than the sound speed, which is ~300 m/s in the lower atmosphere

- **Gravity Waves - these waves carry nearly ALL of the momentum flux and energy (from the linear response) away from typical lower-atmospheric disturbances**

INTERNAL ATMOSPHERIC GRAVITY WAVES AT IONOSPHERIC HEIGHTS[1]

C. O. HINES

ABSTRACT

Irregularities and irregular motions in the upper atmosphere have been detected and studied by a variety of techniques during recent years, but their proper interpretation has yet to be established. It is shown here that many or most of the observational data may be interpreted on the basis of a single physical mechanism, namely, internal atmospheric gravity waves.

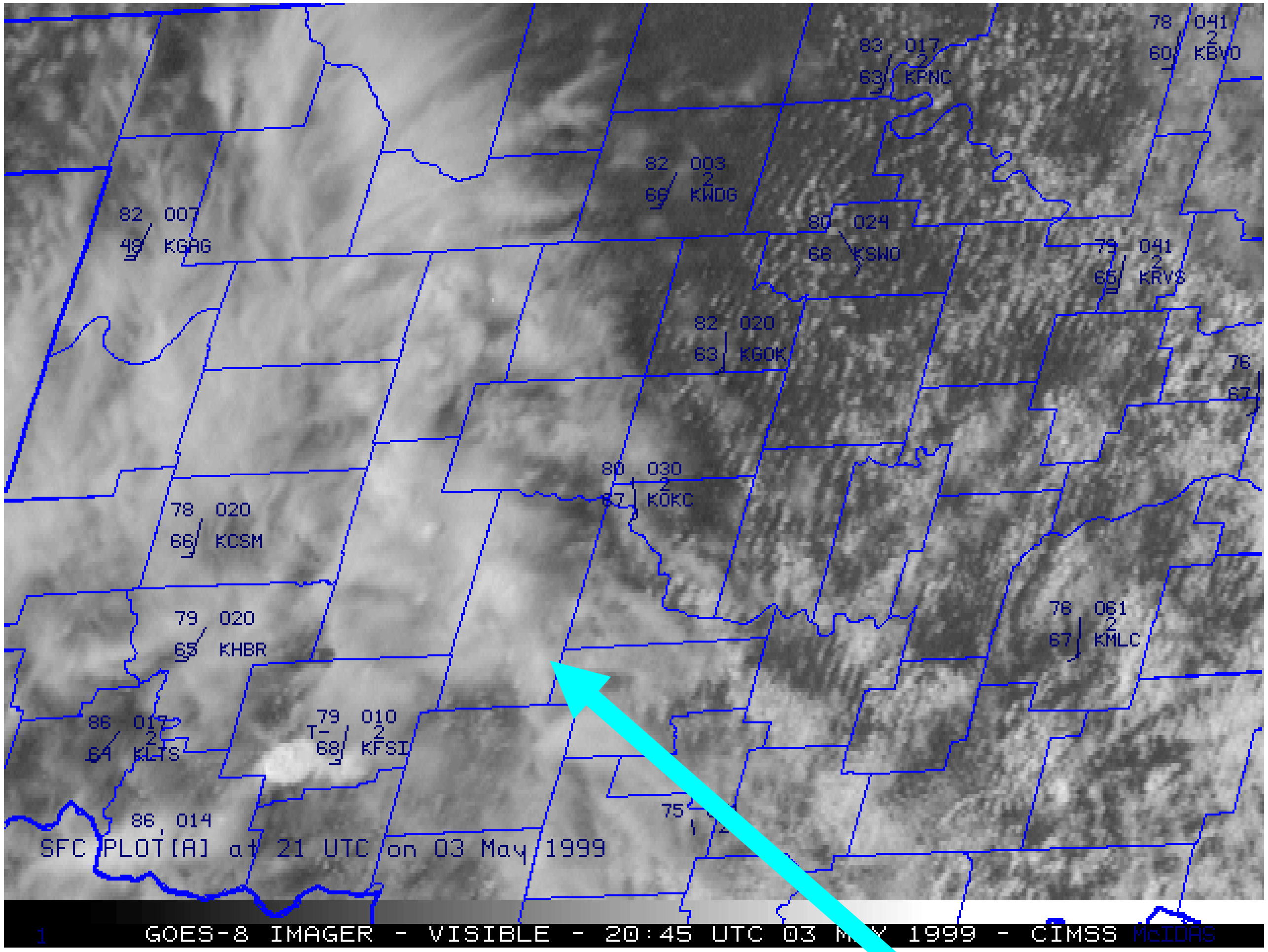
A comprehensive picture is envisaged for the motions normally encountered, in which a spectrum of waves is generated at low levels of the atmosphere and propagated upwards. The propagational effects of amplification, reflection, intermodulation, and dissipation act to change the spectrum continuously with increasing height, and so produce different types of dominant modes at different heights. These changes, coupled with an observational selection in some cases, lead to the various characteristics revealed by the different observing techniques. The generation of abnormal waves locally in the ionosphere appears to be possible, and it seems able to account for unusual motions sometimes observed.

I. INTRODUCTION

Much attention has been devoted in recent years to the detection and measurement of irregular motions in the D, E, and lower F regions of the upper atmosphere, and to the occurrence of irregular density distributions at the same heights. Only tentative interpretations have been put forward until recently, and in the main these have been based on the presumed occurrence of turbulence. It appears, however, that many of the motions and inhomo-

Hines, CJP, 1960

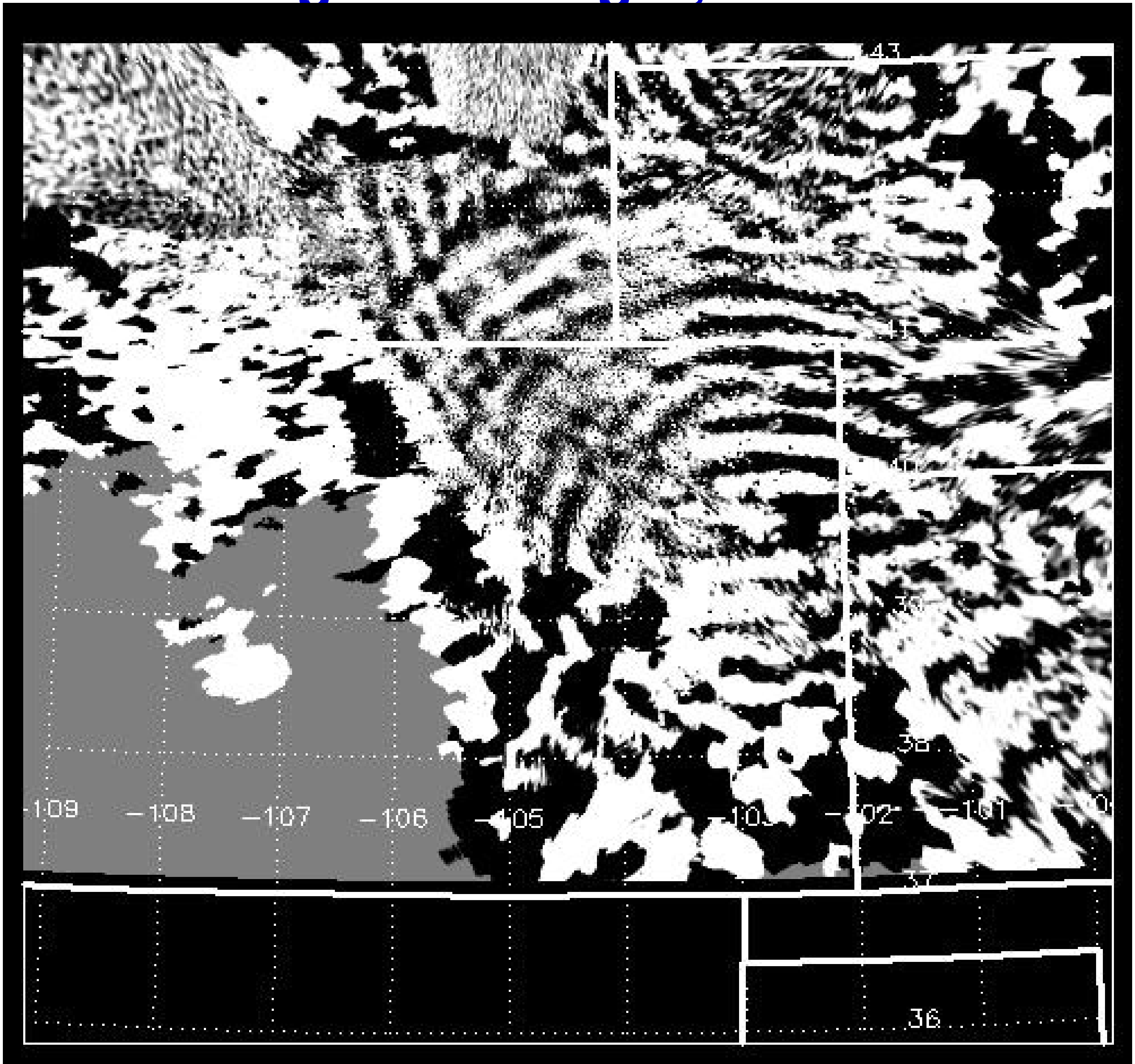
3 May, 1999, near Oklahoma City, Oklahoma



Note the wave-like circular ripples that move out from the overshooting convective plumes

Gravity Waves move upwards and away from the source region, carrying energy and momentum flux

Yucca Ridge OH imager, Colorado 8 Sept, 2005



(Coutesy of Jia Yue, Colorado State)

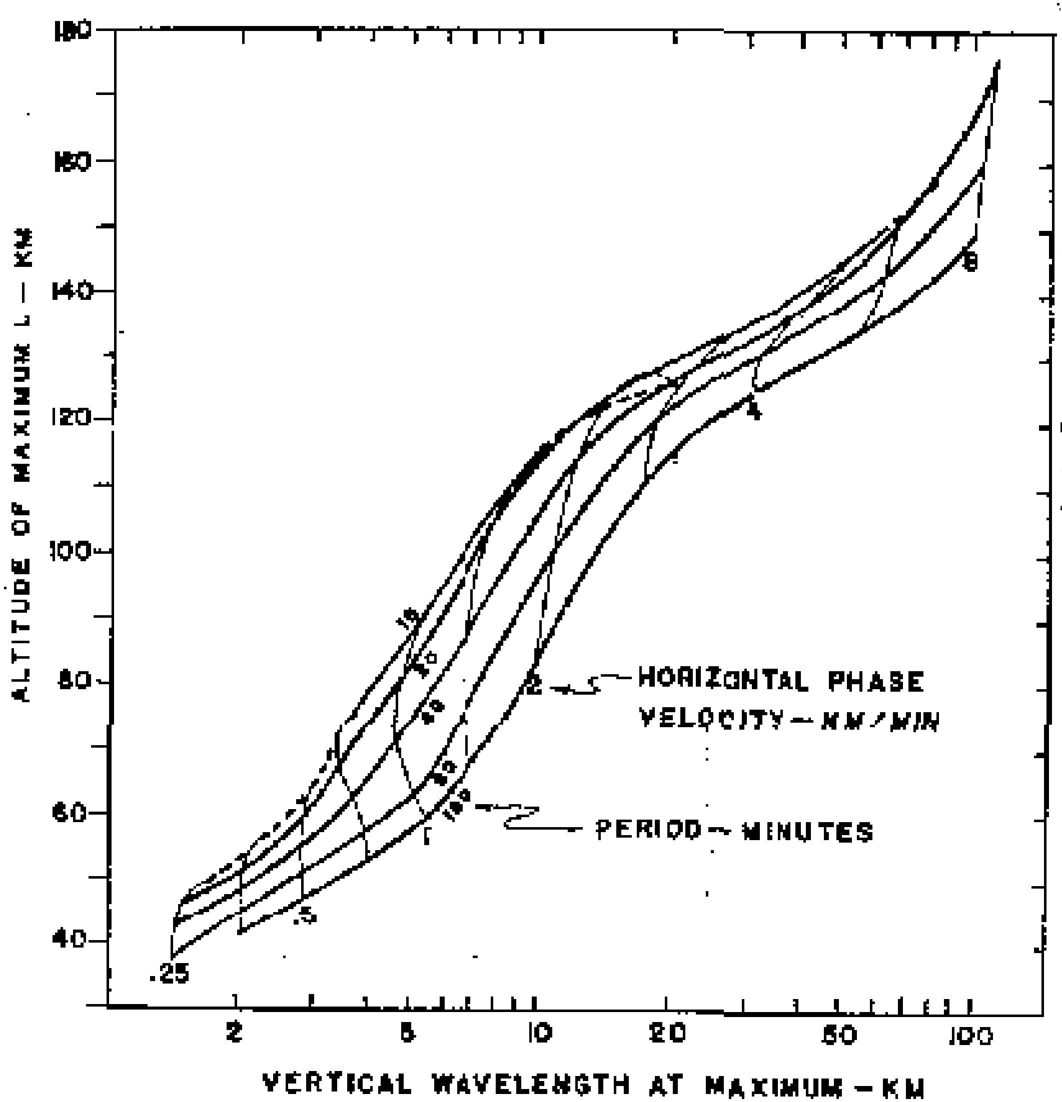
Original question posed by Dave Fritts in 2002:

Can we show via modelling that gravity waves from convection with the right scales and amplitudes are at the bottomside of the F region when plasma instabilities are seeded?

It was well-known that GWs dissipate in the thermosphere

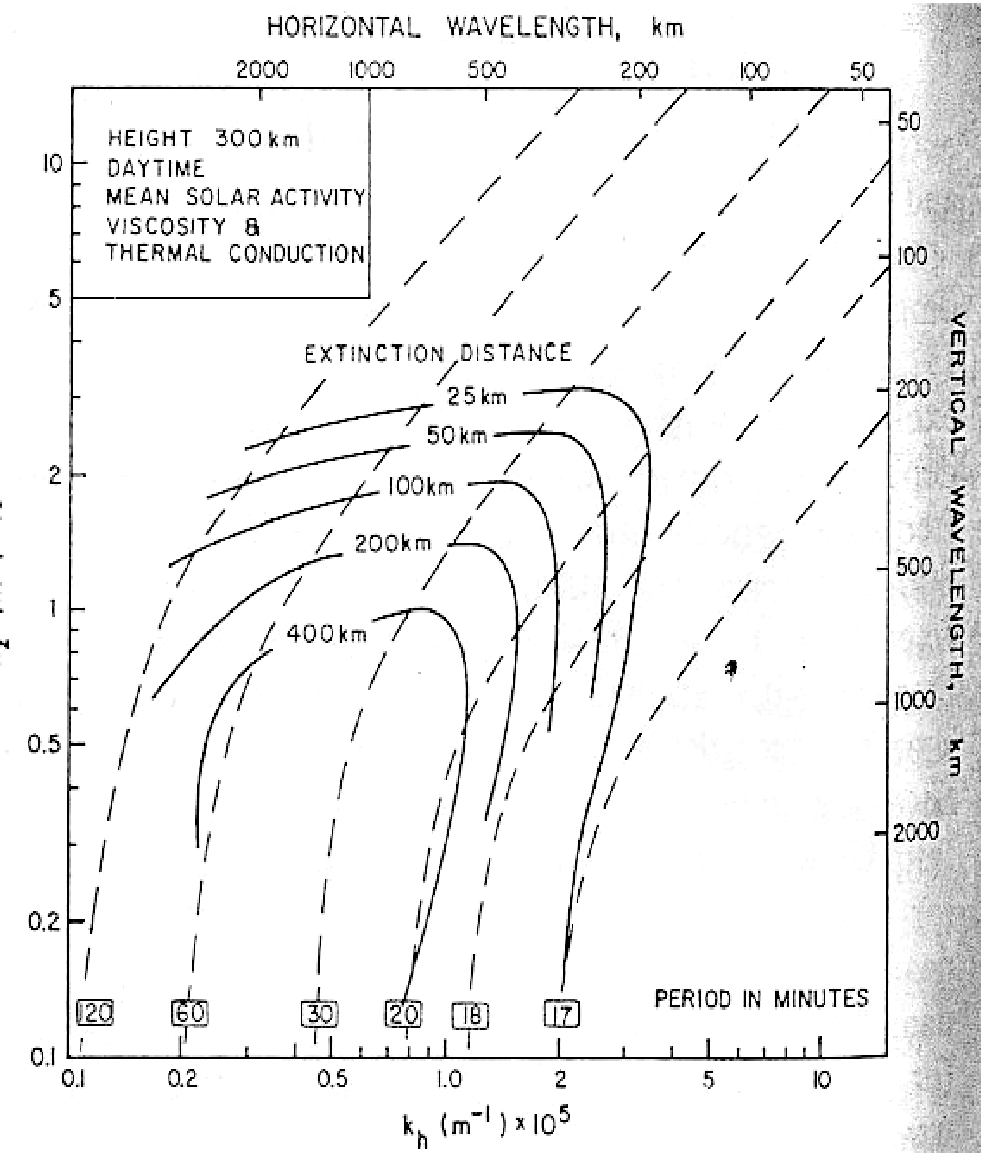
Numerical solutions of GWs dissipating in the thermosphere

Multi-layer approach



(Midgley and Liemohn, JGR, 1966)

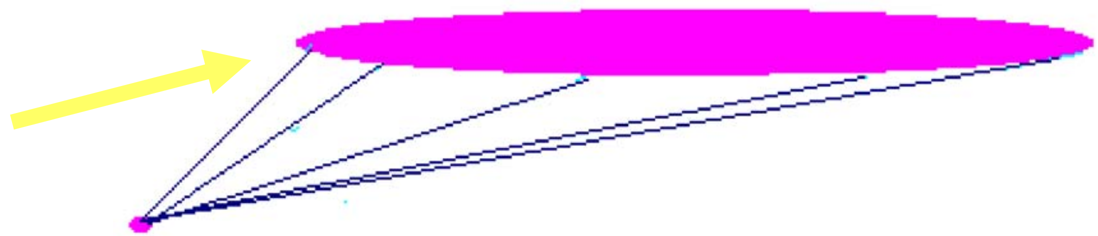
Vertical distance over which amplitude is attenuated by 1/e.



(Yeh etal, AG, 1975)

Due to wave dispersion, a GW packet spreads out to a large volume in thermosphere

Spatial extent of wave packet at z=200-250 km is
~500-1000 km



Convective plume envelope
20 km x 20 km x 10 km

Not possible to simulate both excitation of GWs from convection and propagation/dissipation in thermosphere, with single numerical model

Our solution:

1) Calculate the spectrum of gravity waves using a small-scale, linear model which simulates the updraft of air within a convective plume as a vertical body force,

and

2) Ray-trace these small and medium-scale GWs through realistic winds and temperatures into the thermosphere using a different (ray-trace) model

Why ray-tracing? Because the results from ray-tracing are binned in 4 dimensions (in space and time), the dynamics and influences from both small and large-scale gravity waves can be determined at any altitude of interest

Easier said than done...

- IMPORTANT: Ray-Tracing requires an **<u>analytic</u> gravity wave dispersion relation** which takes into account thermospheric dissipation.

(For gravity waves with periods less than an hour, kinematic viscosity and thermal diffusivity are extremely important, whereas ion drag can be neglected.)

- At the time, only approximate analytic dispersion relations were available which break down when dissipation is strong. Therefore, none of these expressions could be utilized to ray-trace gravity waves.

(e.g., Pitteway and Hines,1963)

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THE VISCOUS DAMPING OF ATMOSPHERIC GRAVITY WAVES

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Received July 25, 1963

ABSTRACT

Dissipation produced by viscous damping and thermal conduction is important in the study of atmospheric gravity waves, which are themselves important in a study of "irregular" motions in the upper atmosphere. The mathematics of this damping is considered in some detail here, and charts are given to assess the effects of viscous damping and thermal conduction at meteor heights in the upper atmosphere. The results of this paper are consistent with the conclusions of an earlier analysis, insofar as the two overlap, and extend the range of conditions considered.

1. INTRODUCTION

Adiabatic waves in an otherwise stationary isothermal atmosphere can be classed as surface waves, whose phase propagation is confined to horizontal directions, and internal waves, which are not similarly constrained; this latter class can be subdivided as acoustic waves and gravity waves according to whether their frequency is above or below a certain forbidden range (Section 2). It is with the latter "internal atmospheric gravity waves" that this paper is concerned. These waves are of interest to studies of the upper atmosphere, for they are believed to be a primary factor in the production of irregular motions and ionization distributions in the D, E, and lower F regions (Hines

(Pitteway and Hines,1963)

The air in our atmosphere is a fluid. The **Navier Stokes compressible, viscous fluid equations** are

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} + \frac{1}{\rho}\nabla p - \mathbf{g} + 2\mathbf{\Omega} \times \mathbf{v} = \mathbf{F} + \frac{1}{\rho}\nabla(\mu\nabla\mathbf{v})$$

conservation of momentum

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla.\mathbf{v} = 0$$

conservation of mass

kinematic viscosity

$$\frac{\mathrm{D}T}{\mathrm{D}t} + (\gamma - 1)T\nabla.\mathbf{v} = J + \frac{1}{\rho}\nabla.\left(\frac{\mu}{\mathrm{Pr}}\nabla T\right)$$

conservation of heat

Thermal diffusivity

ρ: mean mass density of fluid

p: press

v: velocity

F: body force

J: heating

T: temperature

g: gravity

Ω: Earth's rotation

μ: molecular viscosity

Pr: Prandtl number

Assume the background temperature is constant with altitude (i.e., isothermal)

$$\overline{\rho} = \overline{\rho}_0 \mathrm{e}^{-z/H}$$

Density decreases exponentially with altitude

Linearize the fluid variables,

$$\begin{aligned} u &= U + u' \\ v &= V + v' \\ w &= w' \\ \rho &= \overline{\rho} + \rho' \\ T &= \overline{T} + T' \\ p &= \overline{p} + p' \end{aligned}$$

wave solutions

(k,l,m) is the wavenumber

$$\begin{aligned} u' &= \mathrm{e}^{z/2\mathrm{H}} u_0' \mathrm{e}^{i(\omega t - kx - ly - mz)} \\ v' &= \mathrm{e}^{z/2\mathrm{H}} v_0' \mathrm{e}^{i(\omega t - kx - ly - mz)} \\ w' &= \mathrm{e}^{z/2\mathrm{H}} w_0' \mathrm{e}^{i(\omega t - kx - ly - mz)} \\ \rho' &= \mathrm{e}^{-z/2\mathrm{H}} \overline{\rho}_0 \rho_0' \mathrm{e}^{i(\omega t - kx - ly - mz)} \\ p' &= \mathrm{e}^{-z/2\mathrm{H}} \overline{\rho}_0 p_0' \mathrm{e}^{i(\omega t - kx - ly - mz)} \end{aligned}$$

NOTE: Wave amplitude grows exponentially with altitude

Substitute these wave solutions into the Navier Stokes fluid equations.

The resulting complex gravity wave dispersion relation is:

$$-\quad \underbrace{\frac{\omega_I^2}{c_s^2}(\omega_I - i\alpha\nu)^2\left(1-\frac{i\gamma\alpha\nu}{\mathrm{Pr}\ \omega_I}\right)}_{\text{attributable to sound waves (neglect if only want GWs)}} + \underbrace{(\omega_I - i\alpha\nu)\left(\omega_I - \frac{i\alpha\nu}{\mathrm{Pr}}\right)\left(\mathbf{k^2}+\frac{1}{4H^2}\right)}_{\text{gravity waves}} = k_H^2 N^2$$

attributable to sound waves (neglect if only want GWs)

gravity waves

Here, $\alpha \equiv -\mathbf{k}^2 + \dfrac{1}{4\mathrm{H}^2} + \dfrac{im}{\mathrm{H}}$ $\qquad \omega_I = \omega - kU - lV$

How does one solve this complex dispersion relation???

- In all previous studies, m was assumed complex (representing the decay of a wave's amplitude with altitude from dissipation). This makes sense if studying steady-state solutions, but results in an analytic mess. In this case, one only obtains a dispersion relation where dissipation is weak via performing a perturbation expansion to lowest order.

- Instead, Vadas and Fritts (2005) assumed that a wave decays explicitly in time (and implicitly in altitude) by assuming a complex wave frequency ω and a real m. Although these scenarios are equivalent, this assumption results in a real gravity wave dispersion relation and a real decay rate in time accurate when dissipation is strong.

FINAL Anelastic, viscous GW dispersion relation:

$$\boxed{m^2 = \frac{k_H^2 N^2}{\omega_{Ir}^2(1+\delta_+ + \delta^2/\,\mathrm{Pr})}\left[1+\frac{\nu^2}{4\omega_{Ir}^2}\left(\mathrm{k}^2-\frac{1}{4\mathrm{H}^2}\right)^2\frac{(1-\mathrm{Pr}^{-1})^2}{(1+\delta_+/2)^2}\right]^{-1} - k_H^2 - \frac{1}{4\mathrm{H}^2}}$$

$$\mathbf{k} = (k,l,m), \qquad k_H^2 = k^2 + l^2, \qquad \omega_{Ir} = \omega - kU - lV$$

$$\delta = \nu m/\mathrm{H}\omega_{Ir}, \qquad \delta_+ = \delta(1+1/\,\mathrm{Pr}), \qquad \nu_+ = \nu(1+1/\,\mathrm{Pr})$$

Note: δ depends on the intrinsic frequency and the vertical wavenumber!

Wave amplitude decay rate in time:

$1/\omega_{Ii}$, where

$$\omega_{Ii} = -\frac{\nu}{2}\left(\mathbf{k}^2 - \frac{1}{4\mathrm{H}^2}\right)\frac{[1+(1+2\delta)/\,\mathrm{Pr}]}{(1+\delta_+/2)}$$

(Vadas and Fritts, JGR, 2005)

A gravity wave dissipates rapidly above the altitude where

$$c_{g,z}/\omega_{li} \sim H$$

LHS:
vertical distance travelled by the wave over the decay time scale.
Since $c_{g,z}\ \alpha\ \lambda_z\ \omega_{lr}$, and
$|1/\omega_{li}| \sim 2/\nu m^2\ \alpha\ \lambda_z^2$,

LHS $\alpha\ \lambda_z^3\ \omega_{lr}$

RHS:
density scale height

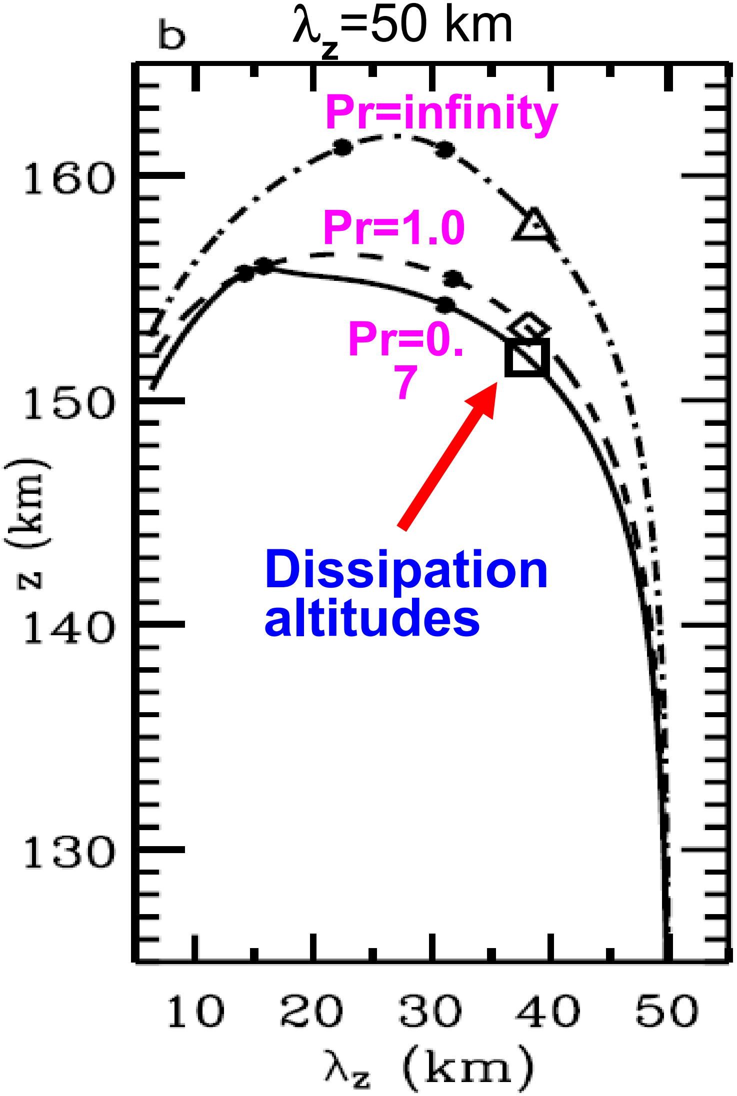
Therefore, **dissipative filtering** removes gravity waves with small λ_z and ω_l at lower altitudes and gravity waves with large λ_z and ω_l at higher altitudes in the thermosphere

When T,U,V are constant, and Pr=1, an exact solution arises:

$$\left(\omega_{Ir} + \frac{m\nu}{H}\right)^2 = \frac{k_H^2 N^2}{\mathbf{k}^2 + 1/4H^2}$$

(In our atmosphere, Pr=0.7)

λ_z=50 km



"ω_{Ir} + $m\nu/H$"

can be thought of as a generalized intrinsic frequency.

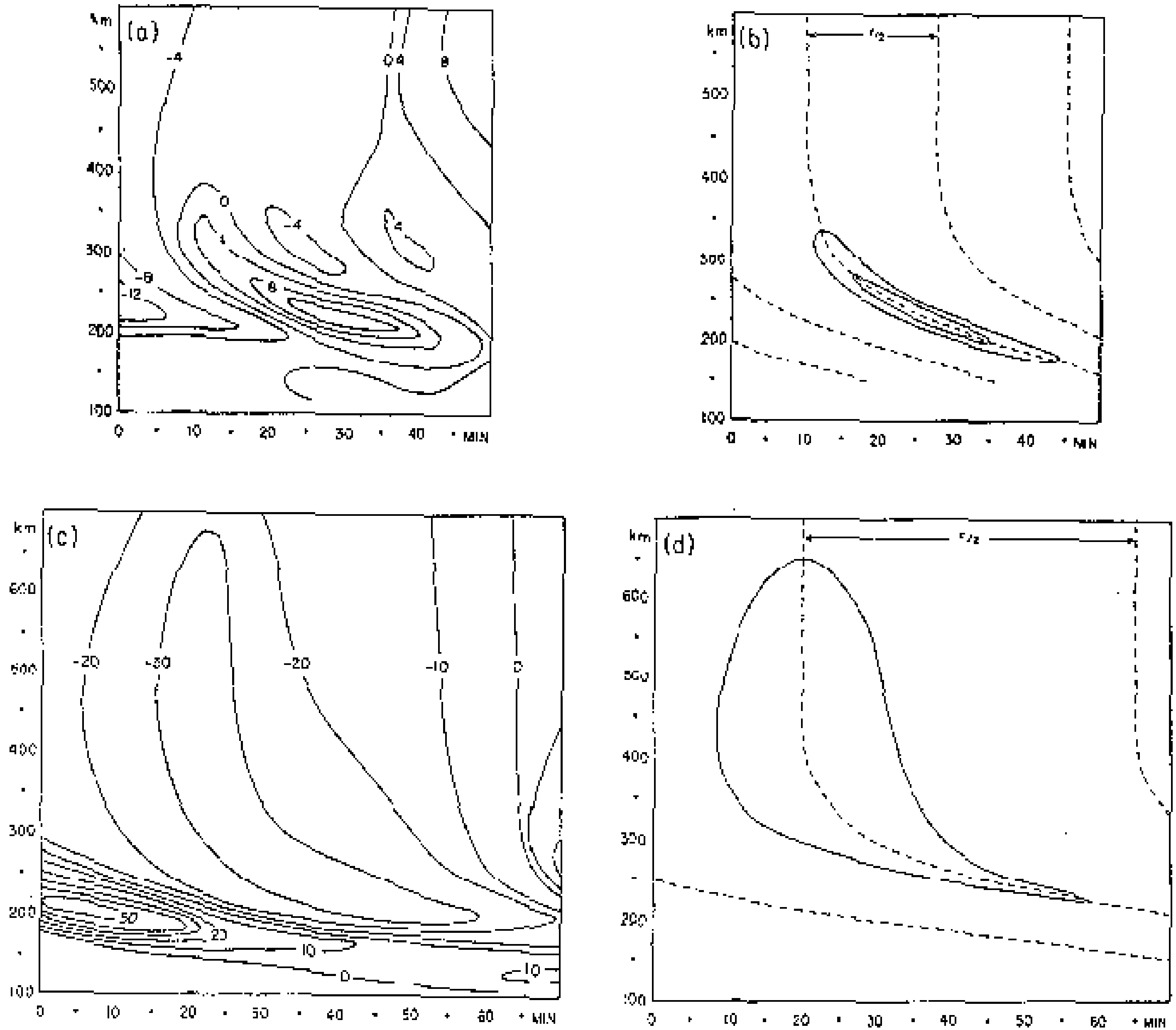
When winds are zero, $\omega_{Ir} = \omega_r$=constant.
m is negative for an upward-propagating GW.
Therefore, LHS decreases with altitude.
Analogous to moving in the direction of the background wind (prior to reaching a critical level, for exp.), this can only occur if λ_z decreases with altitude while dissipating.

(Vadas and Fritts, JGR, 2005)

Hines (1968) used the Pitteway and Hines (1963) dispersion relation to show that these constant ionization perturbation contours from 2 TIDs are the result of GW dissipation, since this dispersion relation predicts that m=0 (or **λ_z=infinity**) when a GW dissipates.

- **However, the Pitteway and Hines dispersion relation is the solution of a perturbation expansion in the kinematic viscosity and thermal diffusivity to lowest order. Therefore, this dispersion relation cannot be used when dissipation is strong.**

- **The Vadas and Fritts dispersion relation is exact when T,U,V are constant, and $\lambda_z<2\pi H$ when dissipation is strong. When a GW dissipates and T and U,V are nearly constant,** λ_z decreases with altitude when it dissipates



(Thome, JGR, 1964)

(Hines, JATP, 1968)

What is ray-tracing?

Propagate a gravity wave upwards and/or downwards in the atmosphere by calculating its changing group-velocity.
Calculate the location, wavenumber **k**=(k_1,k_1,k_1), intrinsic frequency
$\boldsymbol{\omega_{Ir} = \omega_r - k_1 V_1 - k_2 V_2}$, and phase ϕ.
The ground-based frequency, ω_r , is approximately constant.

Mean background wind (**V_1,V_2**)

Lighthill,1978

Wave group velocity, $c_{g_i} = \partial\omega_{Ir}/\partial x_i$

$$\frac{dx_i}{dt} = V_i + \boxed{\frac{\partial\omega_{Ir}}{\partial k_i}} = V_i + c_{g_i}$$

$$\frac{dk_i}{dt} = -k_j \frac{\partial V_j}{\partial x_i} - \boxed{\frac{\partial\omega_{Ir}}{\partial x_i}},$$

Location and wavenumber depends on group velocity, winds, and intrinsic frequency

$$\frac{d\phi}{dt} = \omega_{Ir}$$

Change of wave phase

Analytic derivatives of the GW dispersion relation are **REQUIRED** for ray-tracing

Anelastic, viscous GW dispersion relation:

$$m^2 = \frac{k_H^2 N^2}{\omega_{Ir}^2(1+\delta_+ + \delta^2/\,\mathrm{Pr})}\left[1+\frac{\nu^2}{4\omega_{Ir}^2}\left(\mathrm{k}^2 - \frac{1}{4\mathrm{H}^2}\right)^2 \frac{(1-\mathrm{Pr}^{-1})^2}{(1+\delta_+/2)^2}\right]^{-1} - k_H^2 - \frac{1}{4\mathrm{H}^2}$$

$$\begin{aligned}
\mathbf{k} &= (k,l,m), \quad k_H^2 = k^2 + l^2, \quad \omega_{Ir} = \omega - kU - lV \\
\delta &= \nu m/\mathrm{H}\omega_{Ir}, \quad \delta_+ = \delta(1+1/\,\mathrm{Pr}), \quad \nu_+ = \nu(1+1/\,\mathrm{Pr})
\end{aligned}$$

Take derivatives of the dispersion relation with respect to k_i and x_i, then separate out all pieces on the LHS, and solve for $c_g = \boxed{\dfrac{\partial \omega_{Ir}}{\partial k_i}}$ and $\boxed{\dfrac{\partial \omega_{Ir}}{\partial x_i}}$,

$$c_{g_x} = \frac{k}{\omega_{Ir}\mathcal{B}}\left[\frac{N^2\left(m^2+1/4\mathrm{H}^2\right)}{\left(\mathbf{k}^2+1/4\mathrm{H}^2\right)^2} - \frac{\nu^2}{2}\left(1-Pr^{-1}\right)^2\left(\mathbf{k}^2-\frac{1}{4\mathrm{H}^2}\right)\cdot\frac{\left(1+\delta_+ +\delta^2/Pr\right)}{\left(1+\delta_+/2\right)^2}\right] \tag{C1}$$

Zonal group velocity

$$c_{g_y} = \frac{l}{\omega_{Ir}\mathcal{B}}\left[\frac{N^2\left(m^2+1/4\mathrm{H}^2\right)}{\left(\mathbf{k}^2+1/4\mathrm{H}^2\right)^2} - \frac{\nu^2}{2}\left(1-Pr^{-1}\right)^2\left(\mathbf{k}^2-\frac{1}{4\mathrm{H}^2}\right)\cdot\frac{\left(1+\delta_+ +\delta^2/Pr\right)}{\left(1+\delta_+/2\right)^2}\right] \tag{C2}$$

Meridional group velocity

$$c_{g_z} = \frac{1}{\omega_{Ir}\mathcal{B}}\left\{m\left[-\frac{k_H^2N^2}{\left(\mathbf{k}^2+1/4\mathrm{H}^2\right)^2} - \frac{\nu^2}{2}\left(1-Pr^{-1}\right)^2\left(\mathbf{k}^2-\frac{1}{4\mathrm{H}^2}\right)\cdot\frac{\left(1+\delta_+ +\delta^2/Pr\right)}{\left(1+\delta_+/2\right)^2} + \frac{\nu^4\left(1-Pr^{-1}\right)^4}{16\mathrm{H}^2\omega_{Ir}^2}\frac{\left(\mathbf{k}^2-1/4\mathrm{H}^2\right)^2}{\left(1+\delta_+/2\right)^3} - \frac{\nu^2}{Pr\,\mathrm{H}^2}\right] - \frac{\nu_+\omega_{Ir}}{2\mathrm{H}}\right\} \tag{C3}$$

Vertical group velocity

$$\begin{aligned}\frac{\partial\omega_{Ir}}{\partial x_i} = \frac{1}{\omega_{Ir}\mathcal{B}}\Bigg\{&\frac{k_H^2N}{\left(\mathbf{k}^2+1/4\mathrm{H}^2\right)}\frac{\partial N}{\partial x_i} + \Bigg[\frac{k_H^2N^2}{4\left(\mathbf{k}^2+1/4\mathrm{H}^2\right)^2} - \frac{\nu^2}{8}\left(1-Pr^{-1}\right)^2\left(\mathbf{k}^2-\frac{1}{4\mathrm{H}^2}\right)\frac{\left(1+\delta_+ +\delta^2/Pr\right)}{\left(1+\delta_+/2\right)^2}\\ &- \frac{\delta^2\nu^2\mathrm{H}^2\left(1-Pr^{-1}\right)^4}{16}\frac{\left(\mathbf{k}^2-1/4\mathrm{H}^2\right)^2}{\left(1+\delta_+/2\right)^3} + \frac{\nu_+ m\omega_{Ir}\mathrm{H}}{2} + \frac{\nu^2m^2}{Pr}\Bigg]\left(\mathrm{H}^{-3}\frac{\partial\mathrm{H}}{\partial x_i}\right) + \Bigg[-\frac{\nu\left(1-Pr^{-1}\right)^2}{16}\frac{\left(\mathbf{k}^2-1/4\mathrm{H}^2\right)^2}{\left(1+\delta_+/2\right)^3}\\ &\cdot\left(4+6\delta_+ + \left(1+\frac{10}{Pr}+\frac{1}{Pr^2}\right)\delta^2 + \frac{2\left(1+Pr^{-1}\right)}{Pr}\delta^3\right) - \frac{m\omega_{Ir}\left(1+Pr^{-1}\right)}{2\mathrm{H}} - \frac{\nu m^2}{Pr\,\mathrm{H}^2}\Bigg]\frac{\partial\nu}{\partial x_i}\Bigg\},\end{aligned} \tag{C4}$$

where the denominator is

$$\mathcal{B} = \left[1+\frac{\delta_+}{2}+\frac{\delta^2\nu^2}{16\omega_{Ir}^2}\left(1-Pr^{-1}\right)^4\frac{\left(\mathbf{k}^2-1/4\mathrm{H}^2\right)^2}{\left(1+\delta_+/2\right)^3}\right]. \tag{C5}$$

(please memorize these formulas for the final exam...)

(Vadas and Fritts, JGR,2005)

This new dispersion relation opened the door for coupling studies via ray-tracing GWs excited from lower atmospheric sources into the thermosphere .

Applications of this dissipative dispersion relation briefly reviewed here:

- 1. Ray-trace white-noise GWs into the thermosphere---how does dissipative filtering affect GWs?

- 2. Ray-trace convectively-generated gravity waves into the thermosphere---do the vertically-dependent wave scales agree with observed GW scales?

- 3. Ray-trace convectively-generated gravity waves to the OH airglow layer, and compare with Yucca Ridge data---is the normalization of the convective plume model OK? (i.e., can we trust it to higher altitudes?)

- 4. Determine the neutral response to wave dissipation in the thermosphere from gravity waves from convection

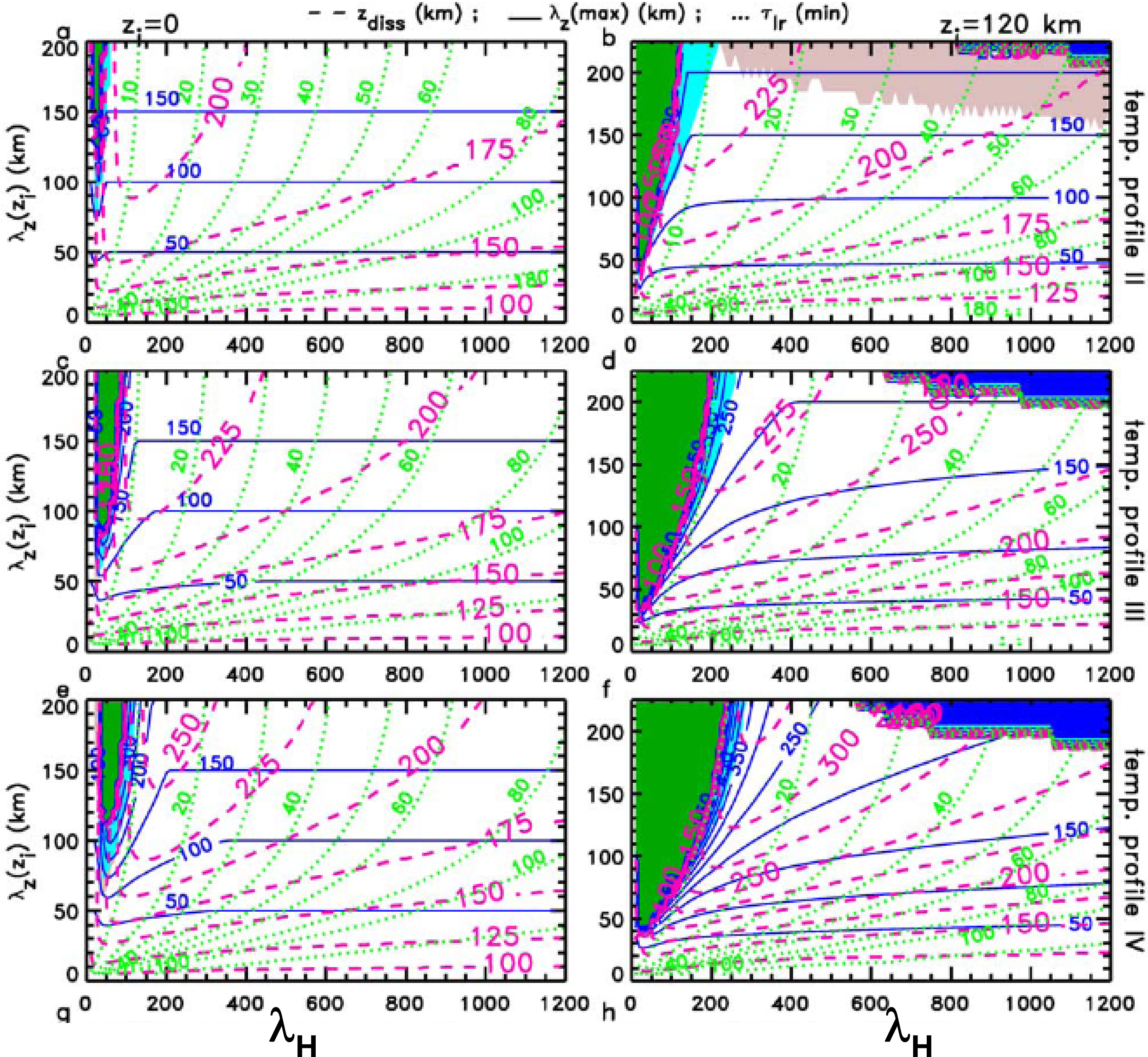
- 5. Extract the vertically-varying, neutral horizontal winds from Poker Flat ISR (AMISR) electron density profiles

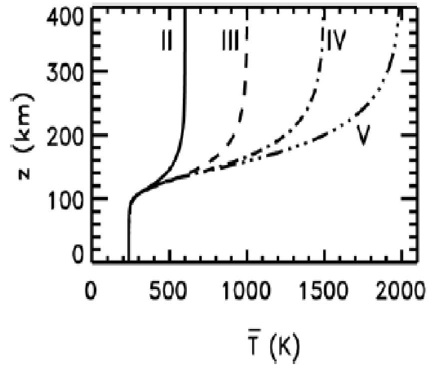
- 1. Ray-trace white-noise GWs into the thermosphere---how does dissipative filtering affect GWs?

“The atmosphere seems to behave like a frequency and height dependent selective filter with respect to gravity waves”,

in reference to the filtering of gravity waves in the thermosphere **---**Volland, JASTP, 491,1969.

Altitudes where GW amplitudes are maximum





T=600K

T=1000K

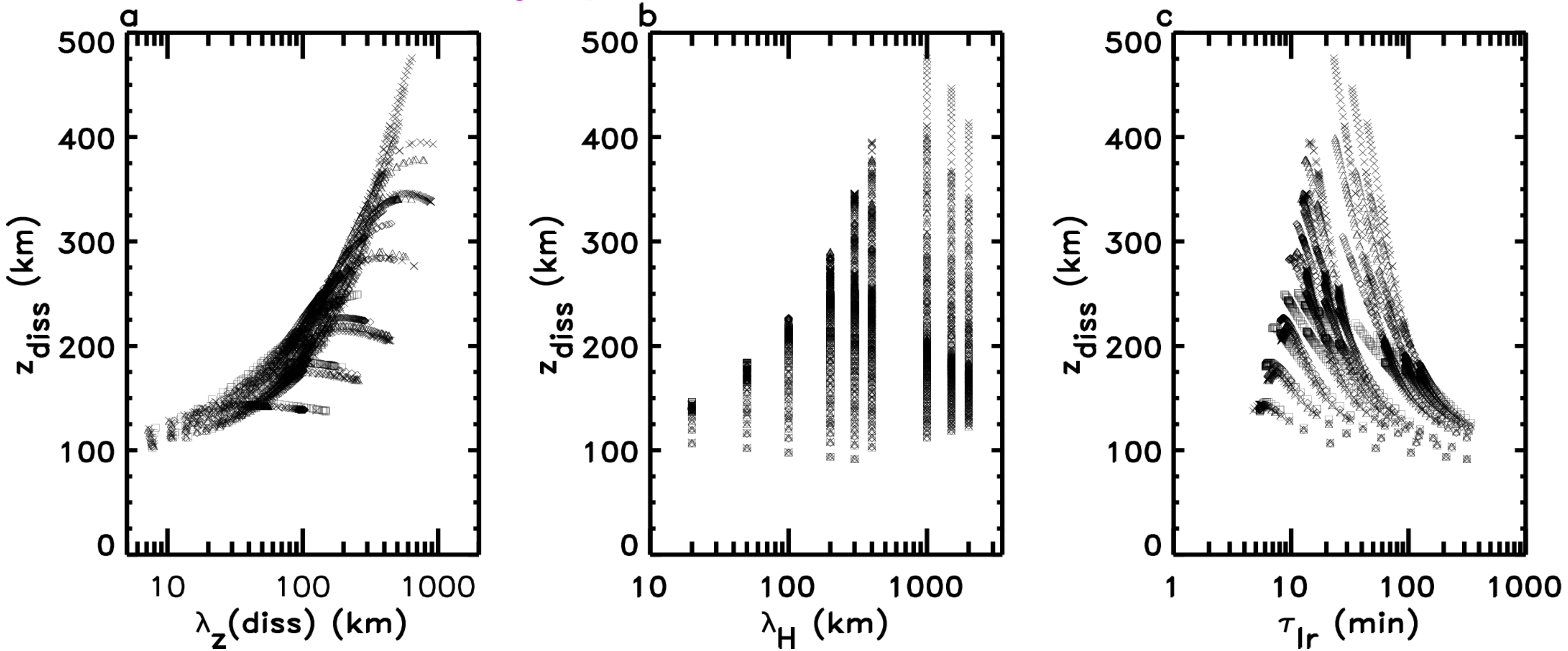
T=1500 K

(Vadas, JGR, 2007)

Dissipation altitudes for "white noise" GWs

Dissipative filtering causes λ_z (for the gravity waves **remaining** in the spectrum) **to increase nearly exponentially with altitude**

λ_H also increases rapidly with altitude, and the wave periods asymptote to 10 - 60 minutes

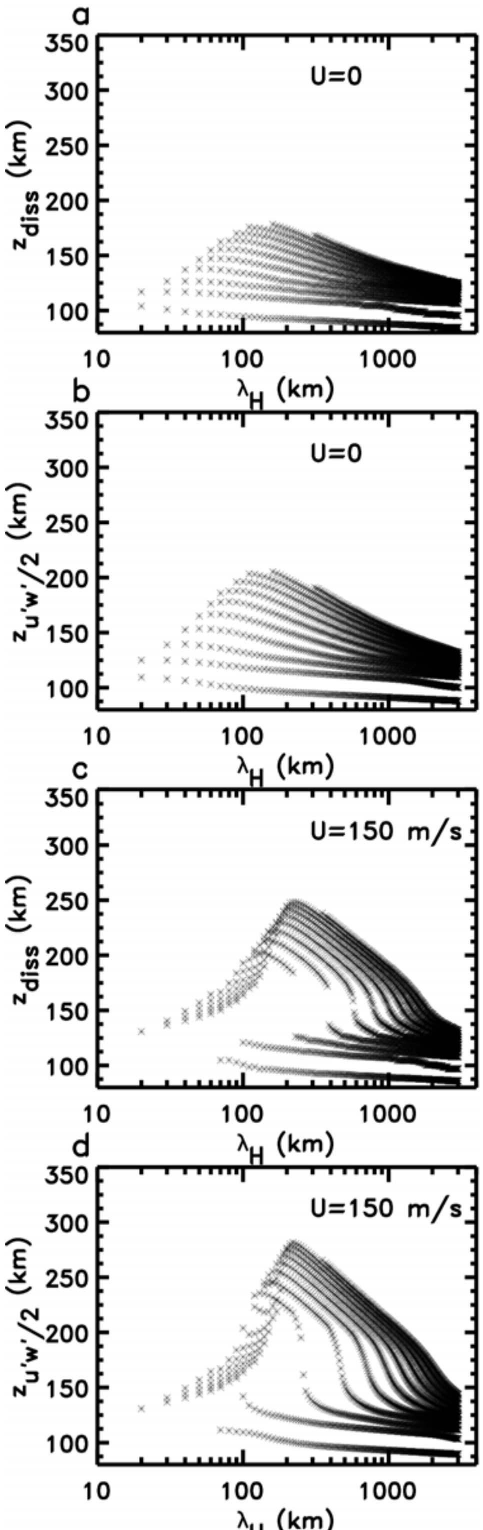


GWs with λ_H ~100-600 km, λ_z~100-125 km and τ ~20-60 minutes propagate well into the F region to z~250 km before dissipating

(Vadas, JGR,2007)

“Satellite-based measurements of gravity wave-induced midlatitude plasma perturbations”

Correlated neutral and plasma density perturbations observed at midlatitudes with the DE2 satellite.



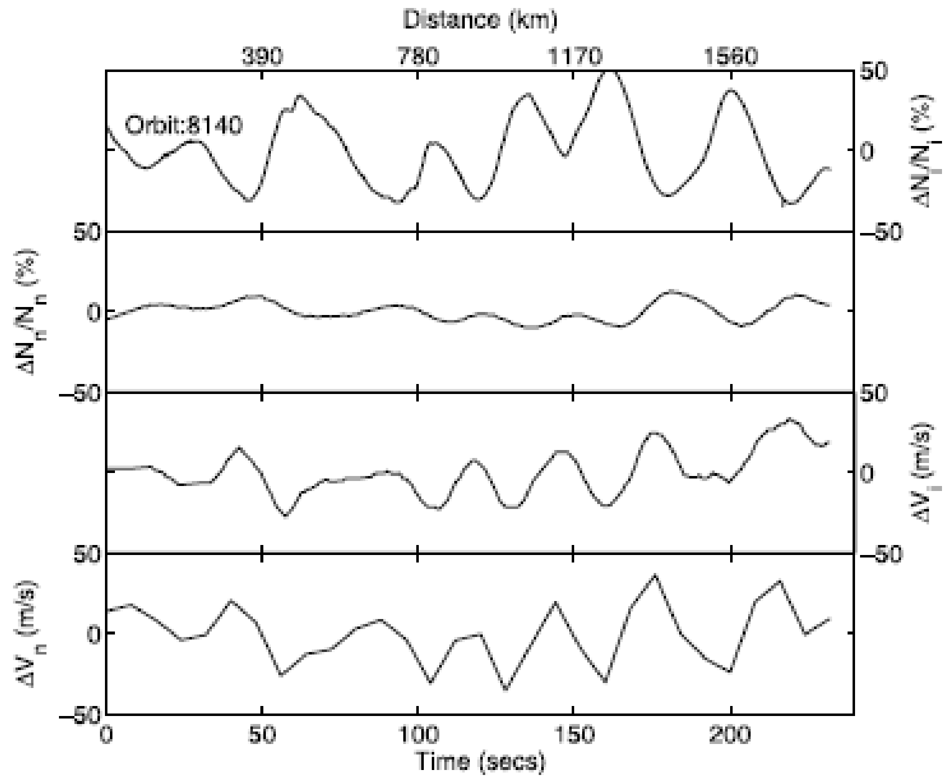
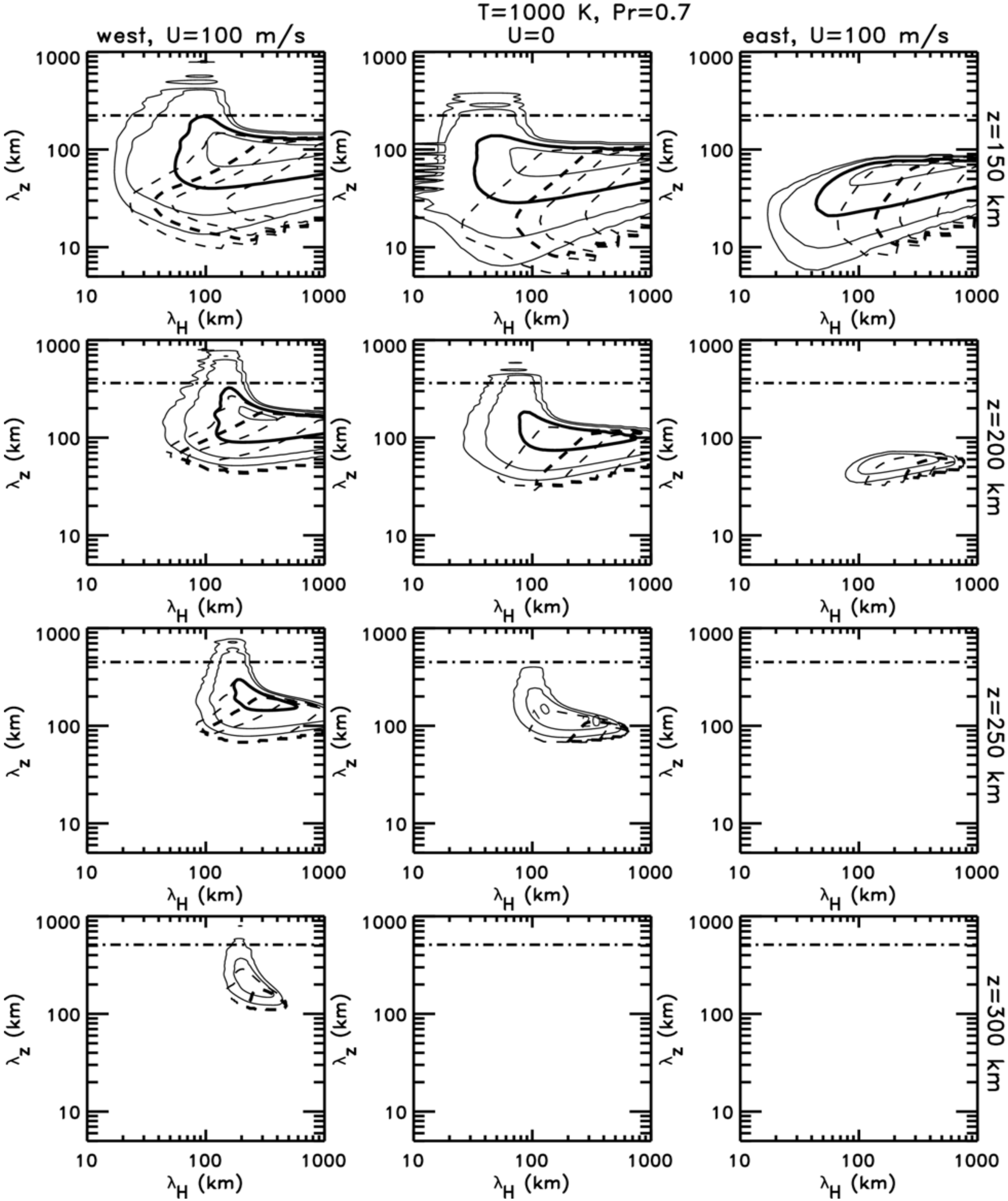


Figure 2. The top two panels show the ion (see Figure 1) and neutral density perturbations for orbit 8140, respectively. The third and fourth panels display the corresponding linearly detrended ion and neutral vertical velocities.

- Only observed the last month of the satellite's life, when the satellite was below 300 km.
- Horizontal wavelengths were 100 km or greater

Theory agrees with data quite well.

(Earle etal, JGR, 2008)



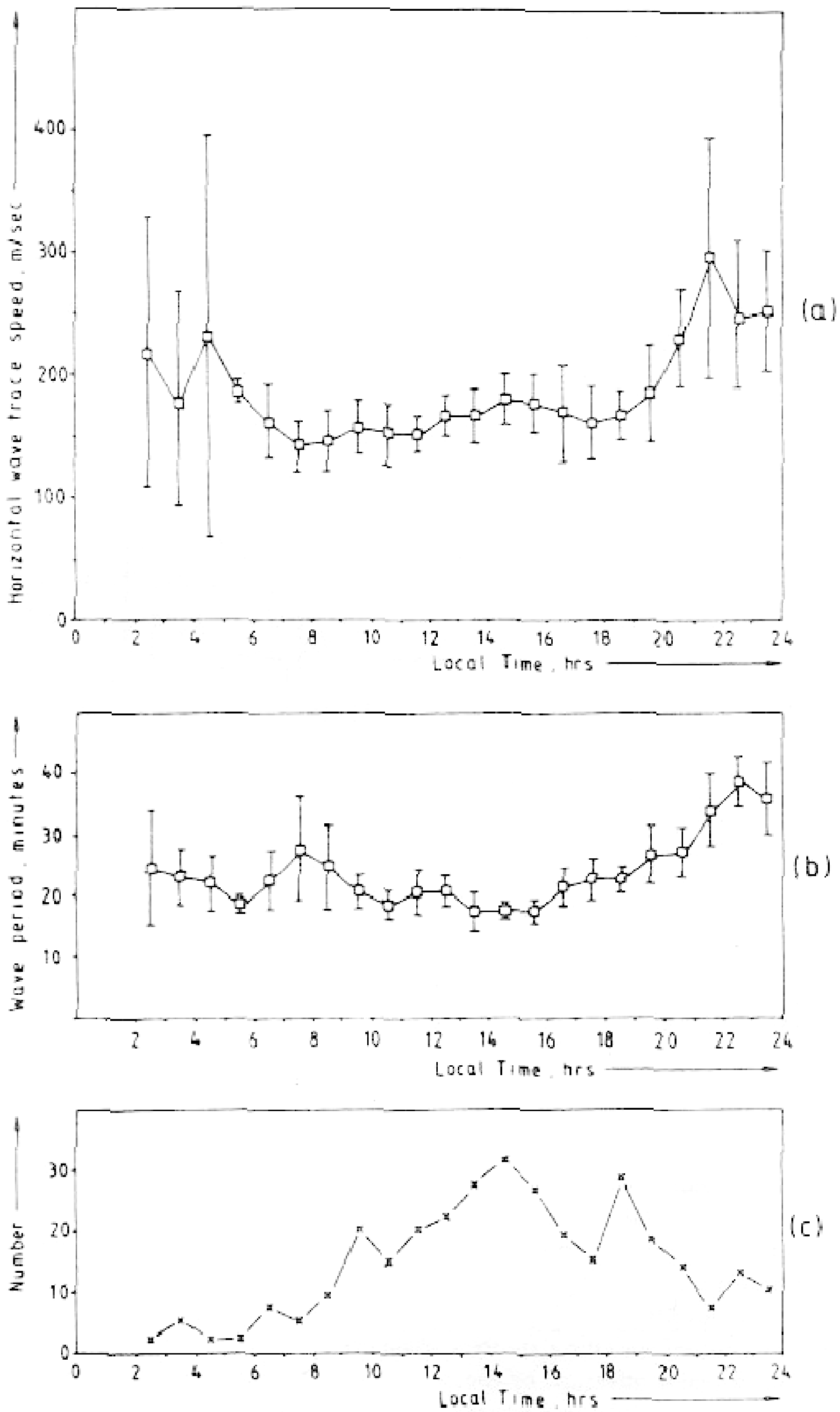
White noise GW spectra from wind, temperature, and dissipative filtering

Those GWs propagating against the wind (west in this example) propagate to the highest altitudes in the thermosphere with λ_H~100-400 km, λ_z~100-300 km, and observed periods of 10-40 min

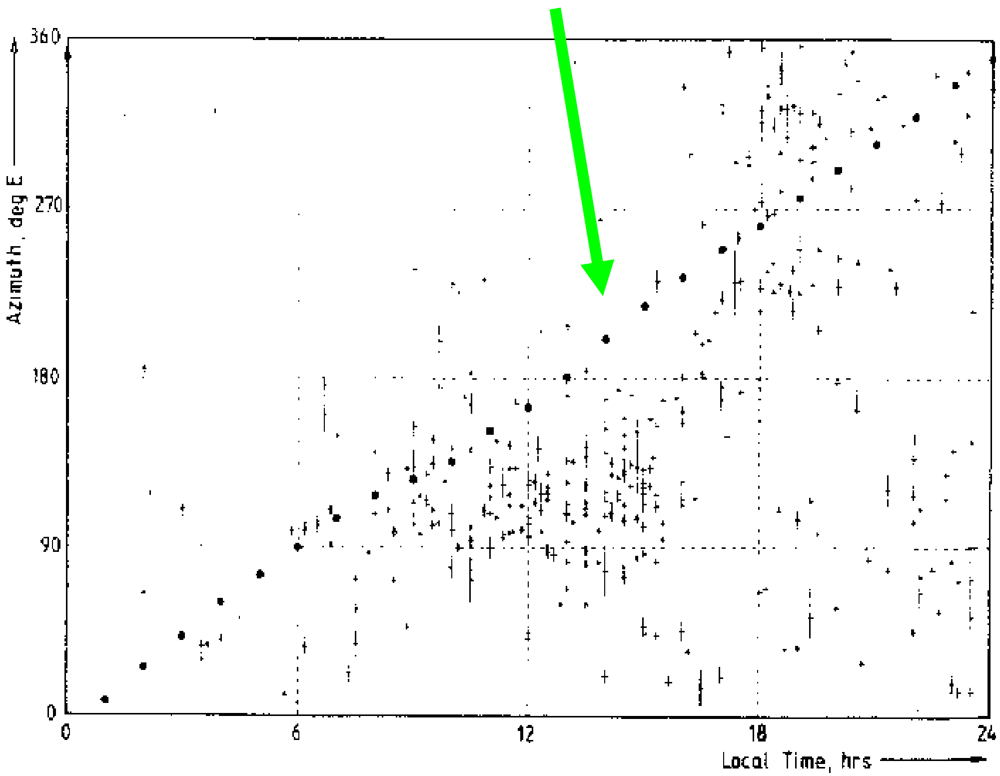
Observed τ=10,20(bold),30, and 60 min

(Fritts and Vadas, 2008, submitted to Annal. Geoph)

Properties of mediun-scale TIDs observed at Leicester, U.K.



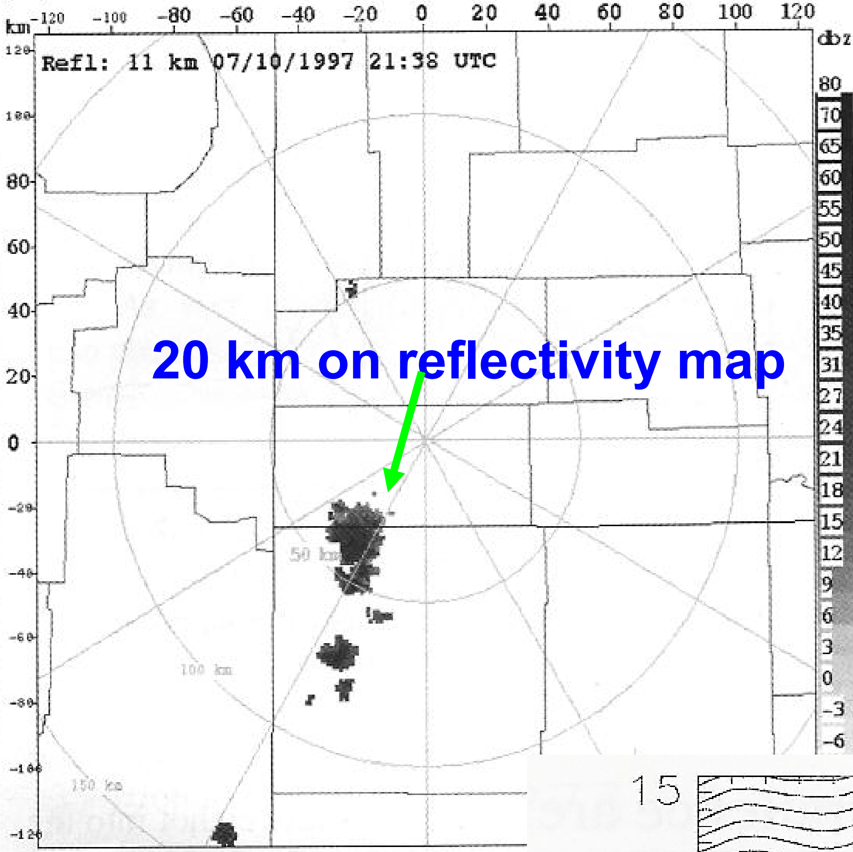
- Periods of 15-40 min
- Phase speeds of 100-250 m/s
- Propagate in all directions, subject to neutral wind
- TIDs tend to propagate opposite to the thermospheric winds



(Waldock and Jones, JATP, 1986)

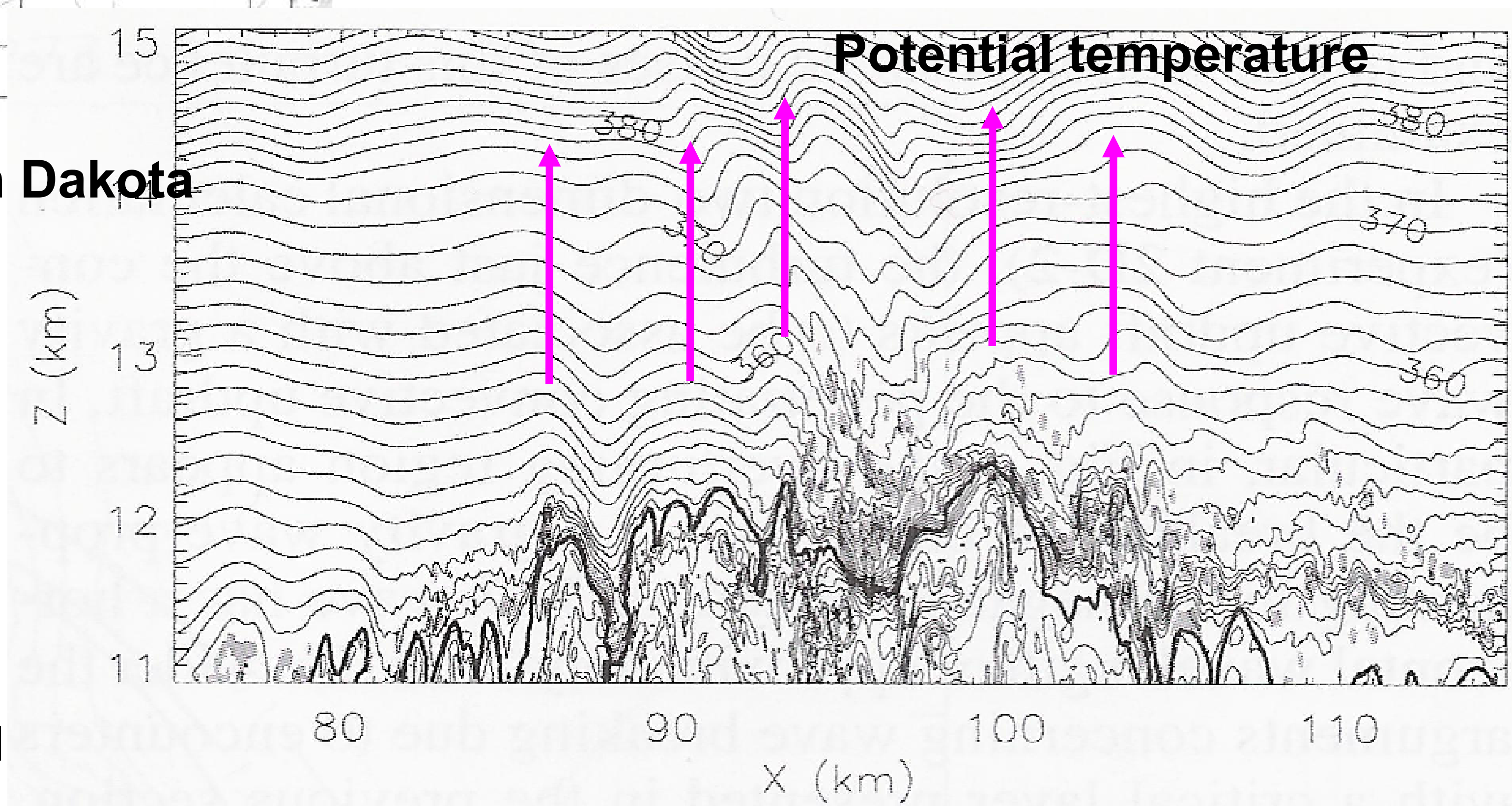
•2. Ray-trace convectively-generated gravity waves into the thermosphere---do the vertically-dependent wave scales agree with observed GW scales?

Full non-linear 2D convection model



10 July, 1997, North Dakota

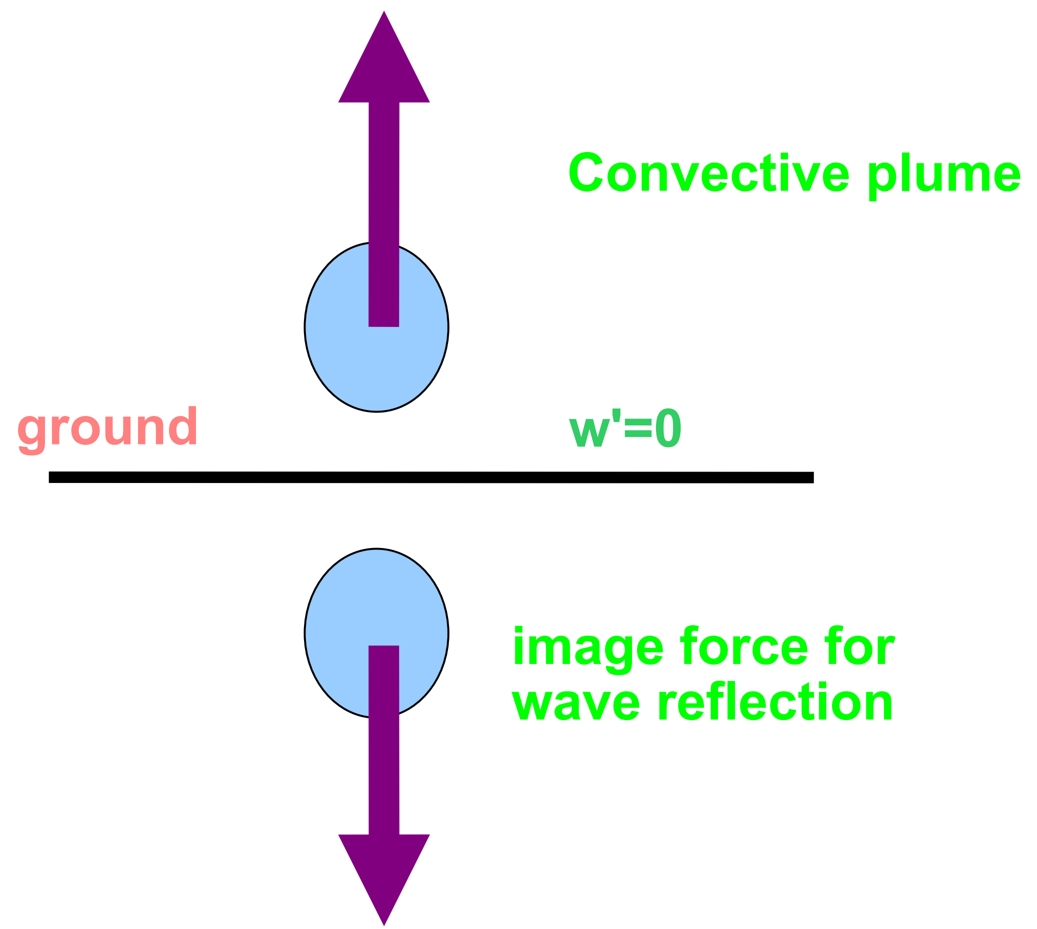
20 km envelope contains lots of smaller convective updrafts (plumes)



(Lane etal, JAS, 2003)

Convective plume model:

- updraft of air is modelled as a vertical body force
- neglect small-scale structure
- retain large-scale envelope of updrafts



(Vadas and Fritts 2004, 2008, submitted to Annal. Geoph)

Fourier-Laplace solutions

Secondary gravity waves are excited from a modelled convective plume via a vertical body force

(Assumes an isothermal, windless environment with constant background density)

$$u(x,y,z,t) = \frac{1}{(2\pi)^3} \int\int\int e^{-ikx-ily-imz} \tilde{u}(k,l,m,t) dk\, dl\, dm (0.1)$$

Post-forcing ($t \geq \sigma_t$) compressible, f-plane solutions to vertical body forcing for mean plus GW only (neglect sound waves):

$$\widetilde{e^{-z/2H}u'} = -\frac{km}{k_H^2}\frac{\omega_{Ir}^2}{N^2}\frac{\hat{a}^2}{\sigma_t \omega_{Ir}(\hat{a}^2 - \omega_{Ir}^2)}\left[1 + i\frac{\gamma-2}{2\gamma H m}\right]\left(\widetilde{e^{-z/2H}F_z}\right)\mathcal{S}$$

$$\widetilde{e^{-z/2H}w'} = \frac{m^2}{k_H^2}\left(\frac{\omega_{Ir}}{N}\right)^4\left[1-\left(\frac{\omega_{Ir}}{N}\right)^2\right]^{-1}\frac{\hat{a}^2}{\sigma_t \omega_{Ir}(\hat{a}^2 - \omega_{Ir}^2)}\left[1+\left(\frac{\gamma-2}{2\gamma H m}\right)^2\right]\left(\widetilde{e^{-z/2H}F_z}\right)\mathcal{S}$$

compressible

(Vadas and Fritts, 2008b, in preparation)

Post-forcing ($t \geq \sigma_t$) Boussinesq, incompressible, f-plane solutions to vertical body forcing are

$$\widetilde{u'} = -\frac{km}{k_H^2}\frac{\omega_{Ir}^2}{N^2}\frac{\hat{a}^2}{\sigma_t \omega_{Ir}(\hat{a}^2 - \omega_{Ir}^2)}\widetilde{F_z}\mathcal{S}$$

$$\widetilde{w'} = \left(\frac{\omega_{Ir}}{N}\right)^2\frac{\hat{a}^2}{\sigma_t \omega_{Ir}(\hat{a}^2 - \omega_{Ir}^2)}\widetilde{F_z}\mathcal{S}$$

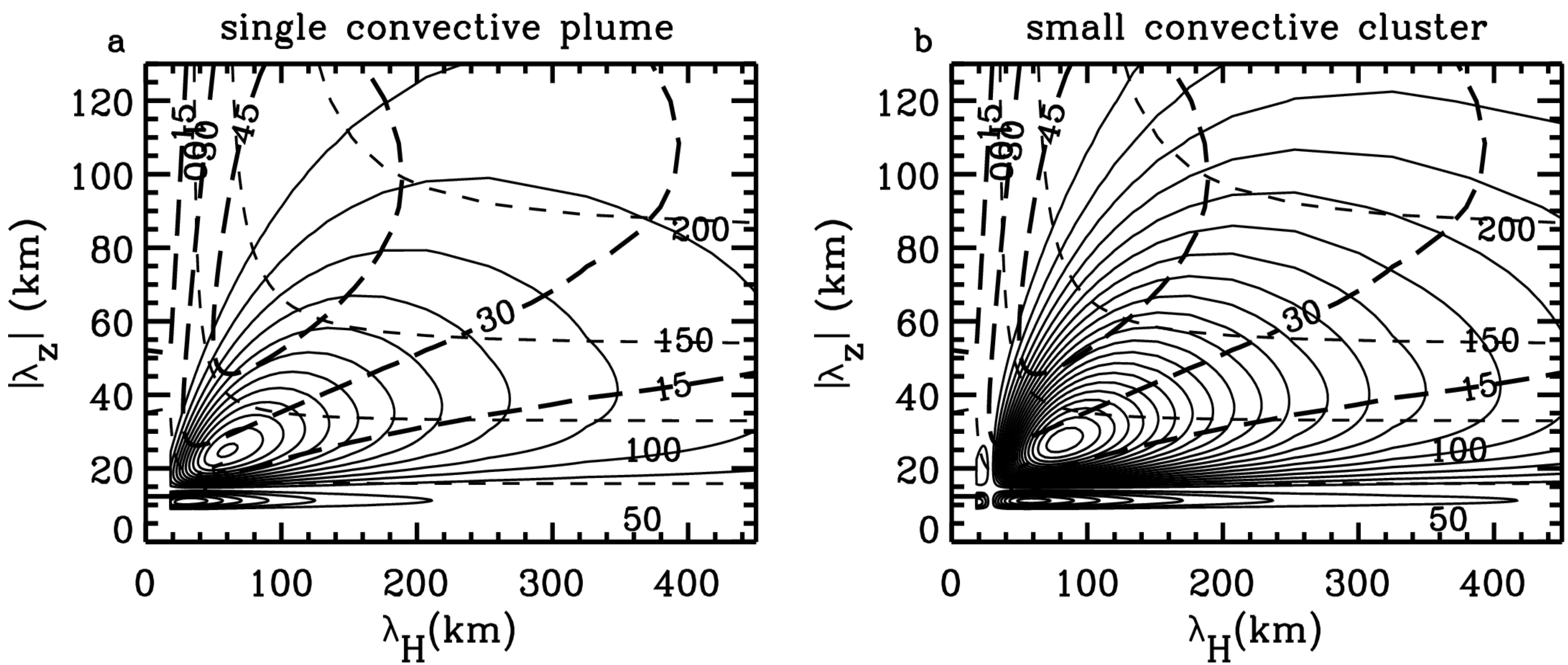
Here, $\mathcal{S} \equiv \sin\omega t + \sin\omega(\sigma_t - t)$, which equals $\mathcal{S} = \omega\sigma_t\cos\omega t$ for fast (or impulsive) forcings.

Boussinesq

(Vadas and Fritts, JAS, 2001)

GW spectra excited from single and multiple convective plumes:

Vertical wavelengths



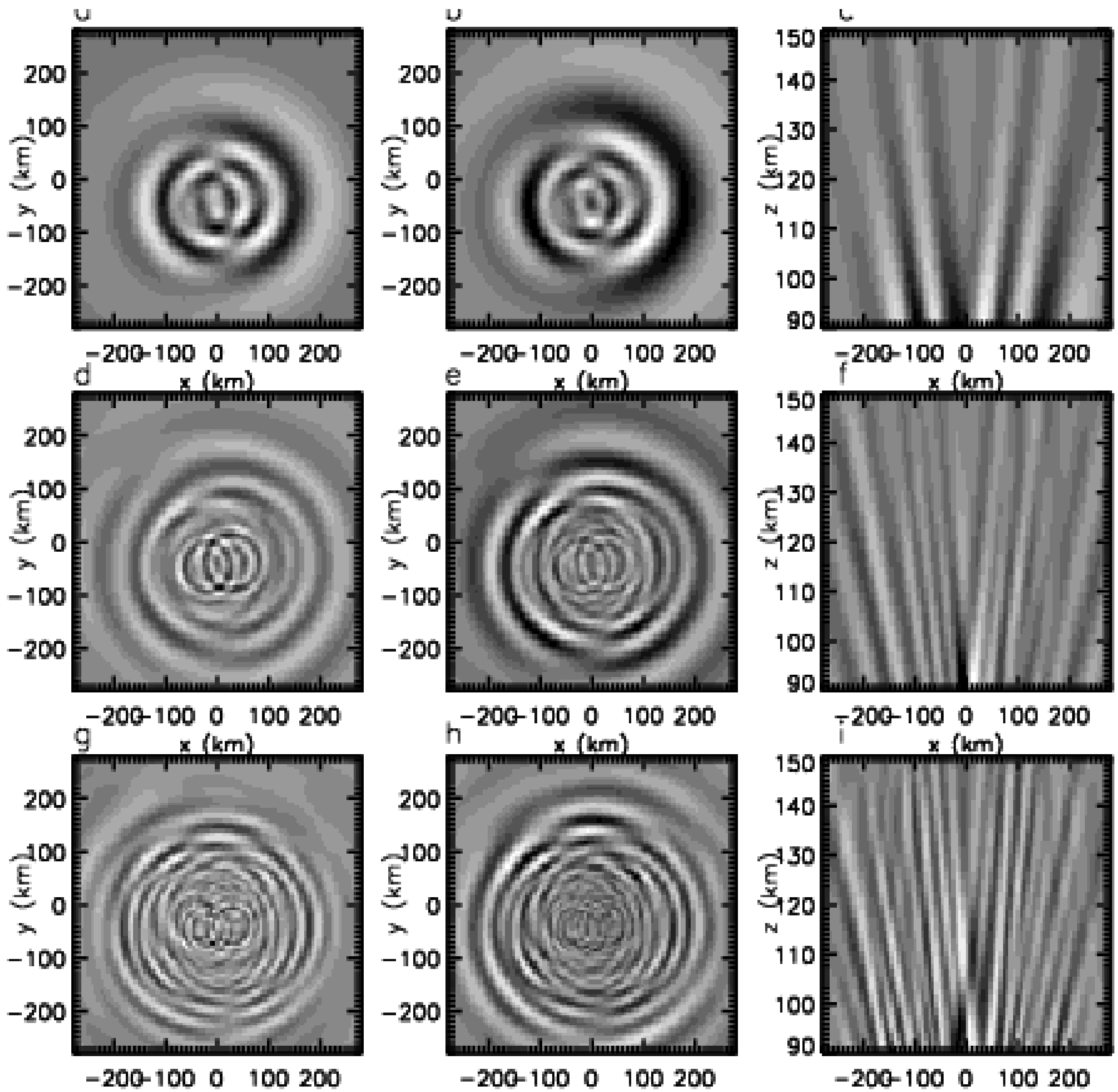
Horizontal wavelengths

Horizontal wavelengths

(Vadas and Fritts, 2008), submitted to Annal. Geoph)

Modeled GWs from mesoscale convective complexes (MCCs)

z=90 km (mesopause)



X (km) X (km) X (km)

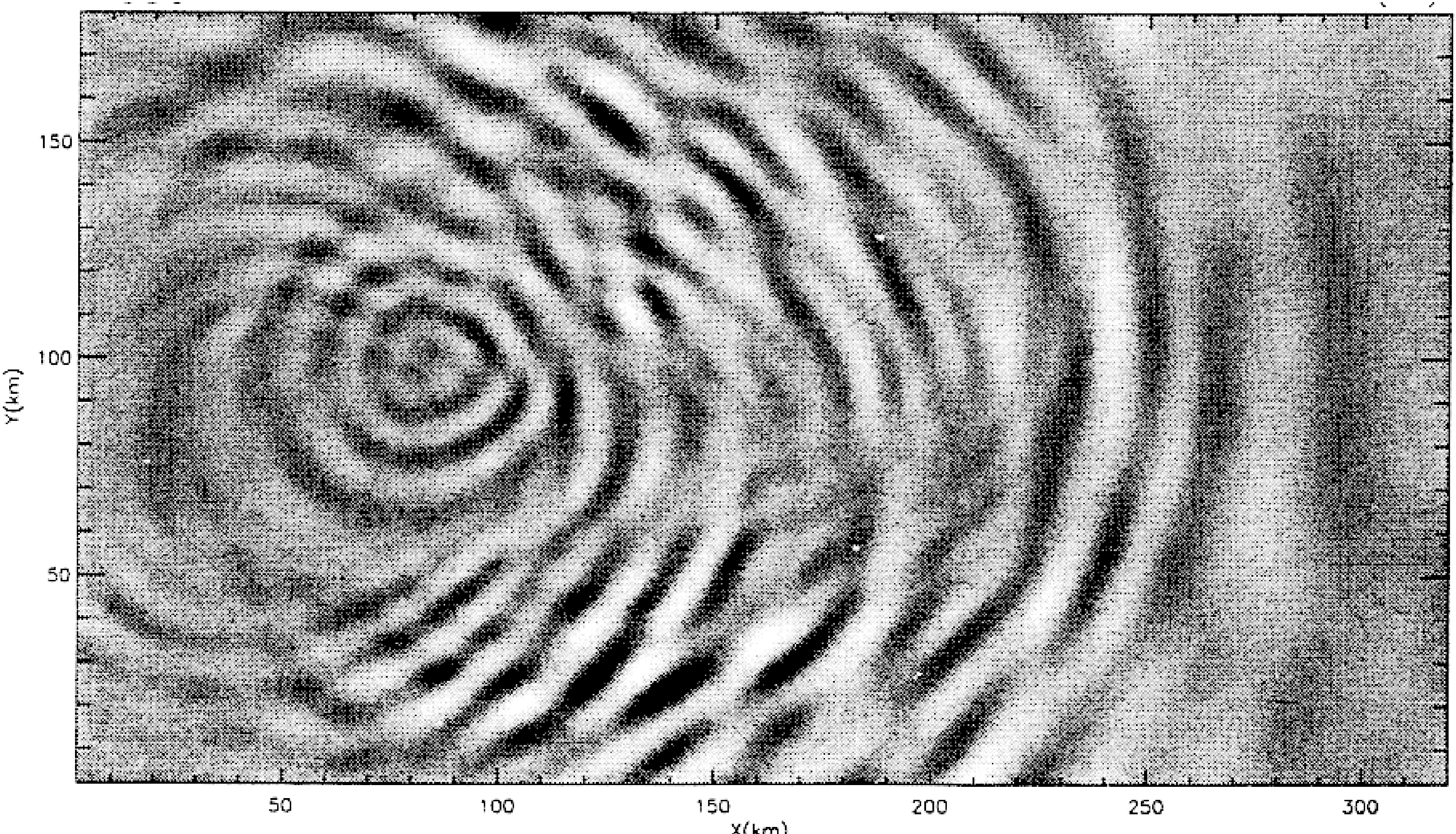
- **Gravity wave modelling (Piani etal, 2000; Lane etal, 2001, 2003, Horinouchi etal, 2002)**

- **Observations of concentric GWs (Taylor and Hapgood, 1988; Dewan etal, 1998; Sentman etal, 2003; Suzuki etal, 2006)**

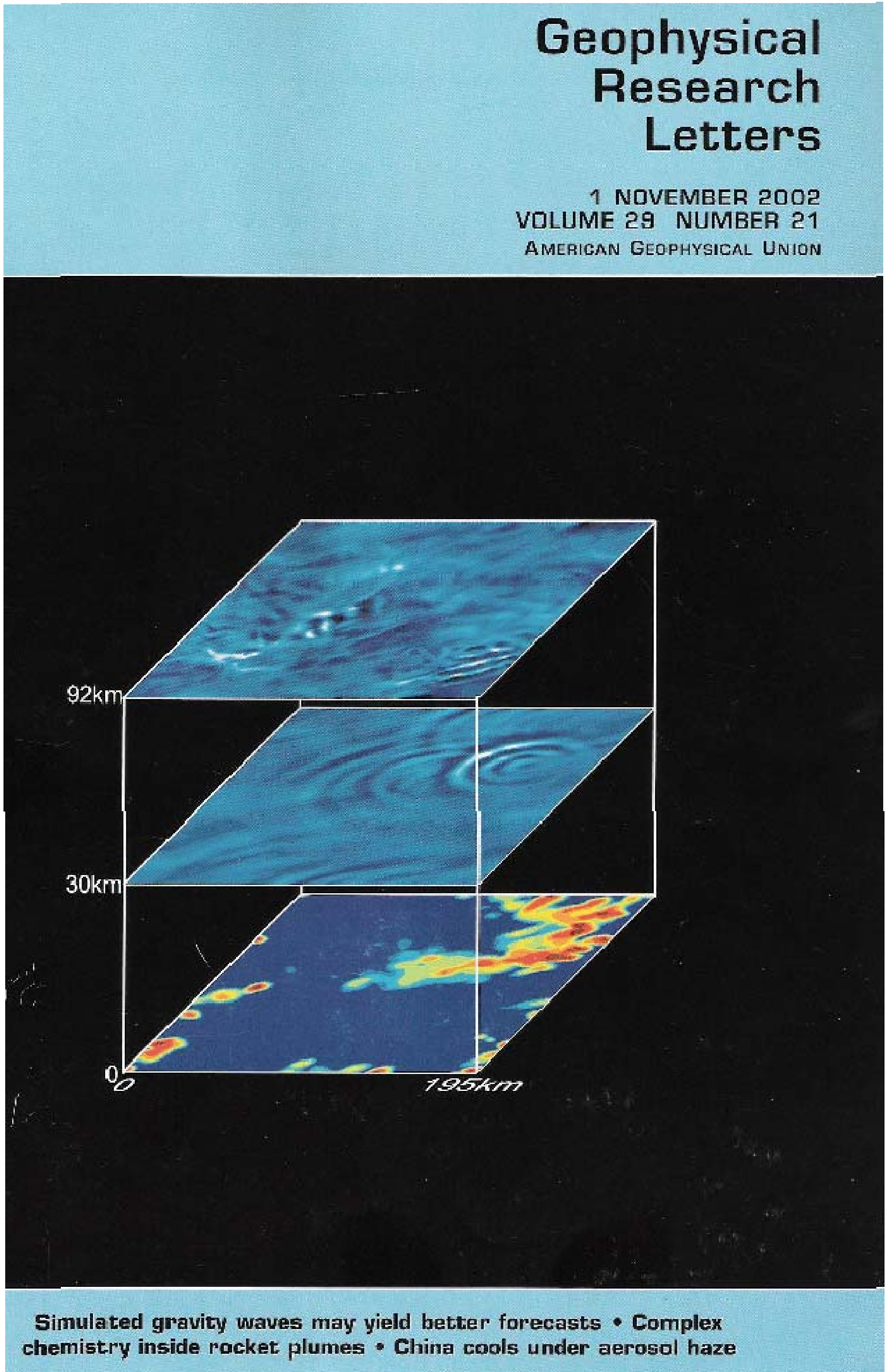
(Vadas and Fritts, JASTP, 2004)

Full non-linear 3D convection model

z=40 km

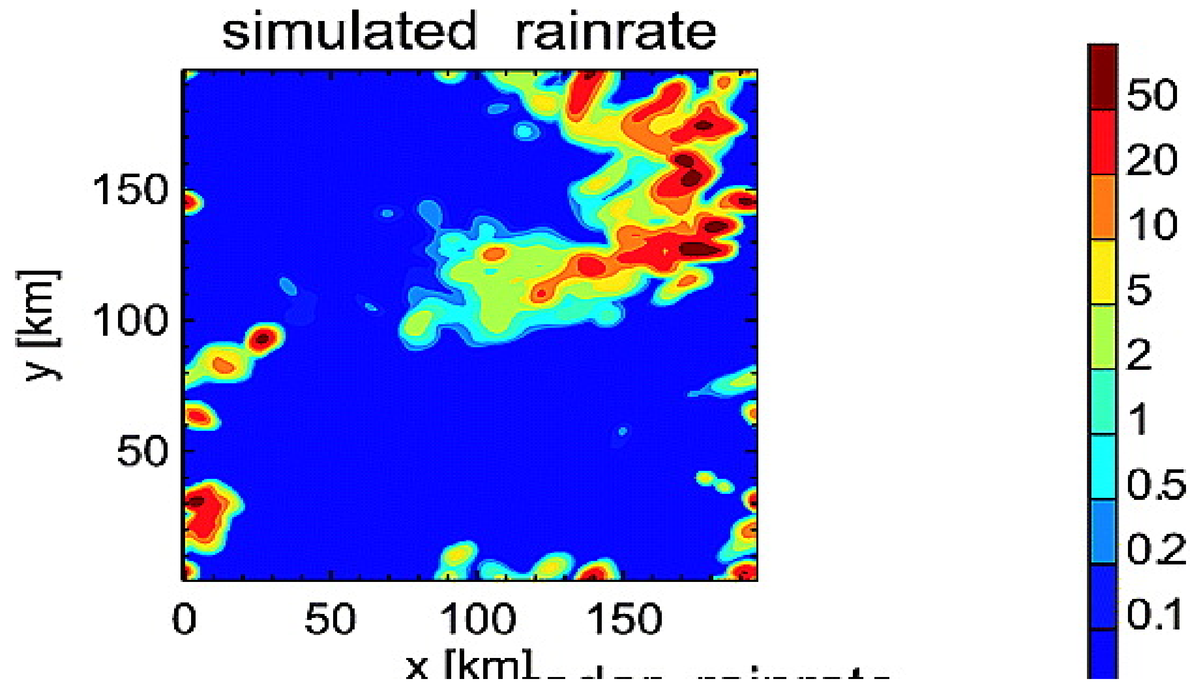


(Lane etal, JAS, 2001)



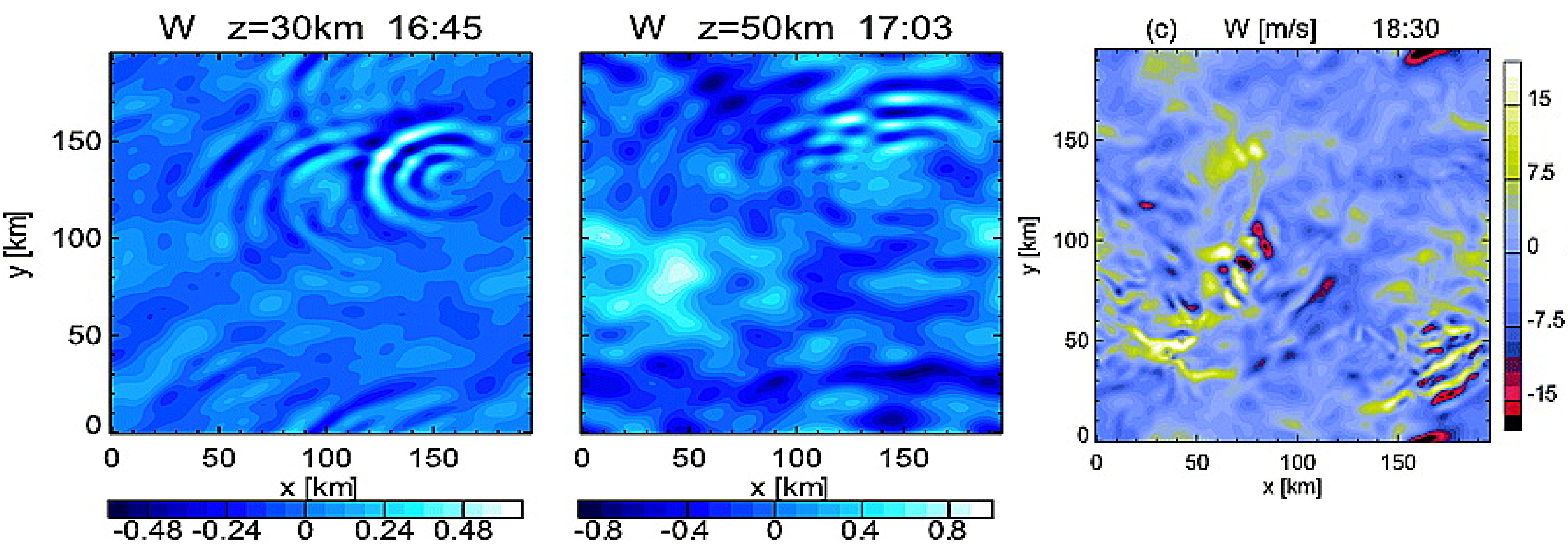
Horinouchi et al, GRL, 2002

Horinouchi et al, GRL, 2002

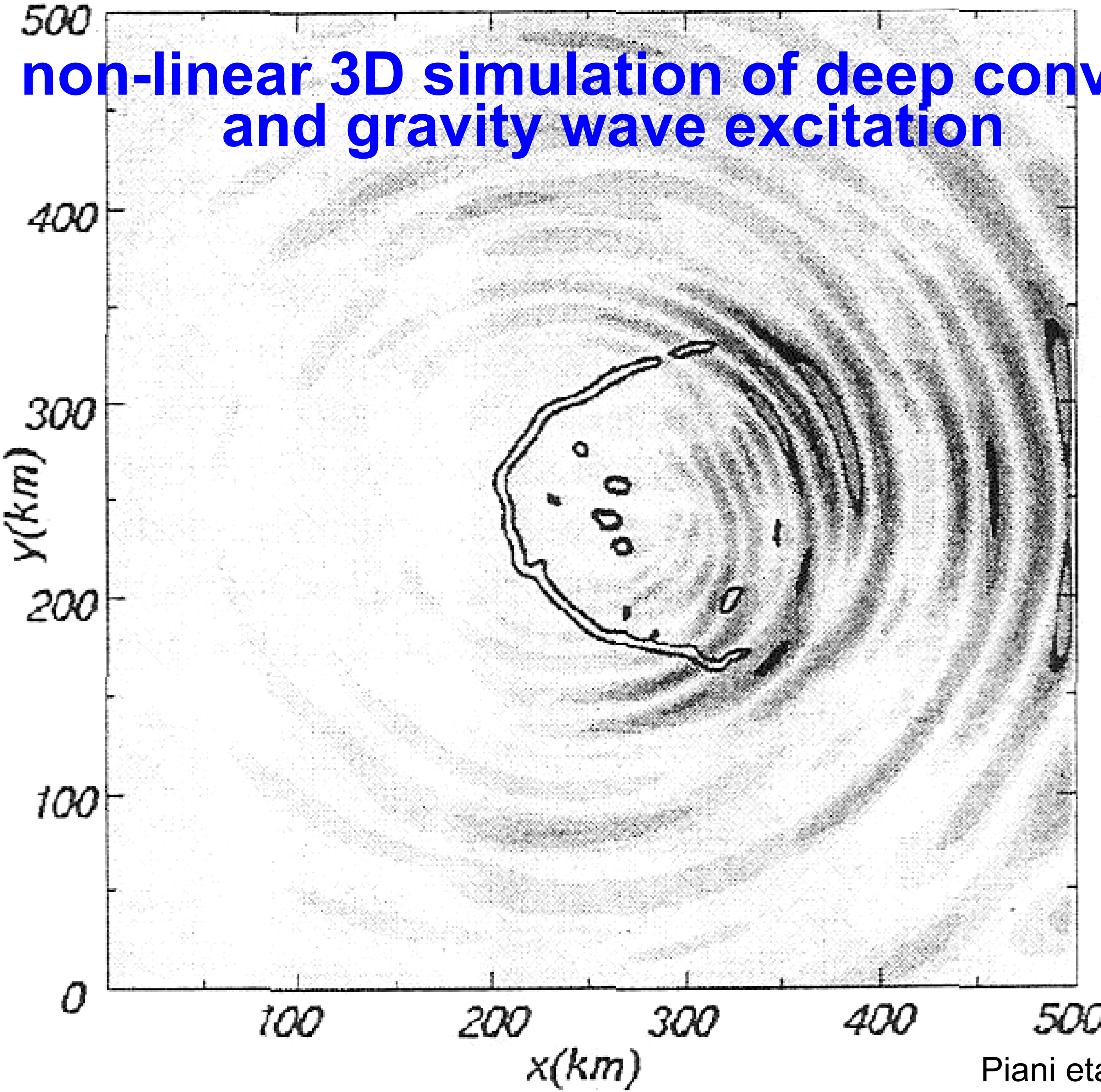


Vertical velocity

z=92 km



Full non-linear 3D simulation of deep convection and gravity wave excitation



Vertical velocity. Wind filtering from the easterly phase of the QBO

Piani etal, JAS, 2000

Advantage of Vadas and Fritts (2004) convective plume model and ray tracing over nonlinear simulations:

- It is much faster (takes only days to run a thunderstorm on a standard desktop machine (versus weeks or months on a super computer for a nonlinear simulation)

- Medium-scale GWs with tiny initial amplitudes (but which are the only GWs left in the F region after dissipative filtering) can be accurately simulated

- There is no need for an upper radiation condition or sponge layer, allowing for computations up to z=500 km

Our strategy:

- 1) Calculate the wave amplitudes and scales that are excited via convective plumes using this Fourier-Laplace idealistic model,

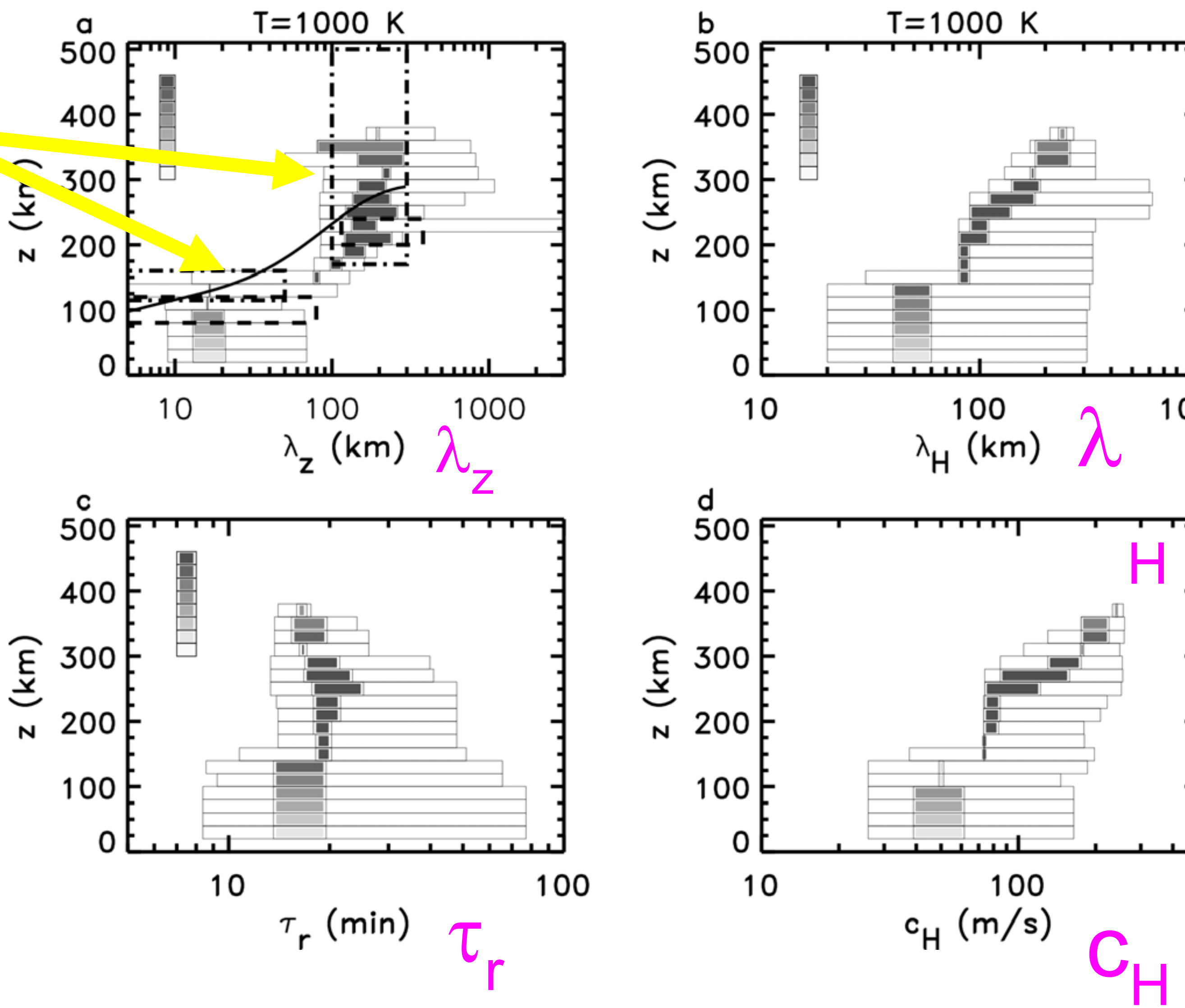
- 2) Embed these excited GWs into the frame of the wind at the tropopause,

- 3) Ray-trace these GWs into the **mesosphere** and **thermosphere** through variable winds and temperatures using the anelastic gravity wave dispersion relation

Application #1: compare wave scales from convection with measured wave scales

Altitude dependence of GWs from a single convective plume which propagate through a lower thermospheric shear

Djuth et al and Oliver et al results



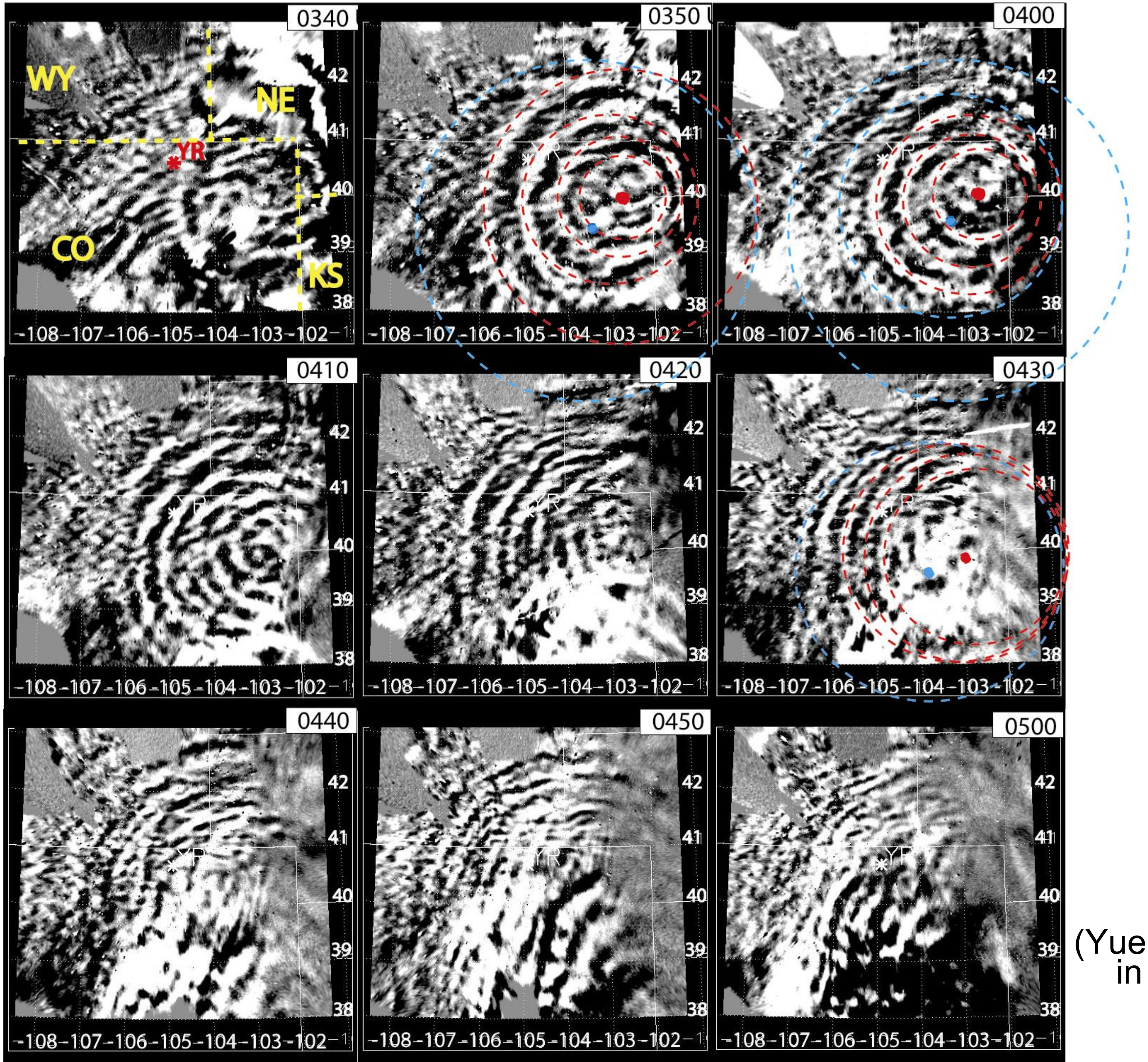
Theory agrees with data quite well.

At the highest altitudes, only those medium-scale GWs remain

(Vadas, JGR,2007)

- 3. Ray-trace convectively-generated gravity waves to the OH airglow layer, and compare with Yucca Ridge data---is the normalization of the convective plume model OK? (i.e., can we trust it to higher altitudes?)

Yucca Ridge OH imager, Colorado 11 May, 2004



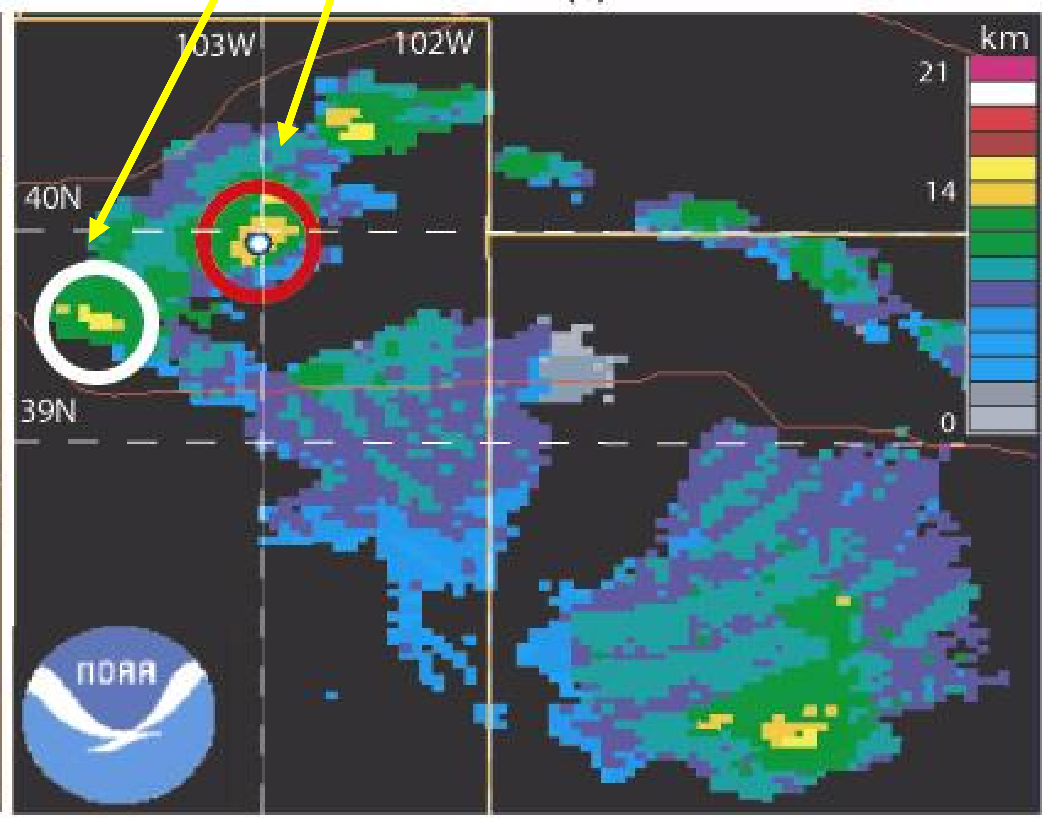
(Yue et al, 2008, in preparation)

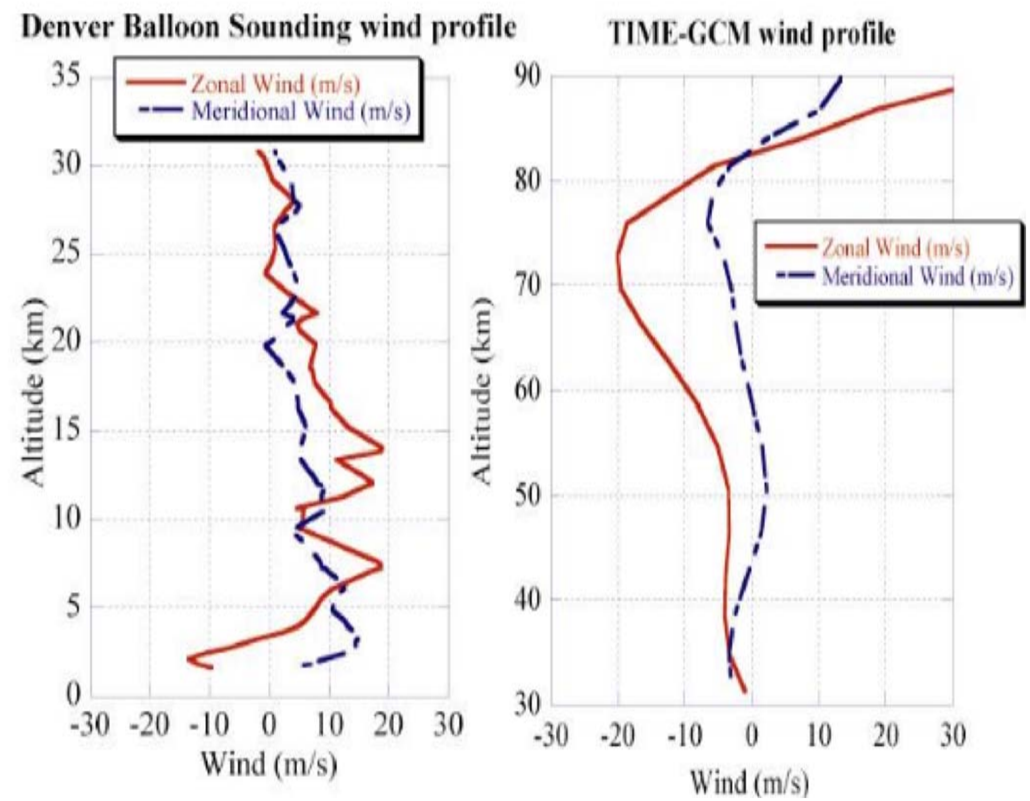
NOAA NEXRAD Doppler radar at 3:05 UT: identify regions of convective overshoot

These concentric rings were centered on 2 convective plumes separated by ~ 100 km

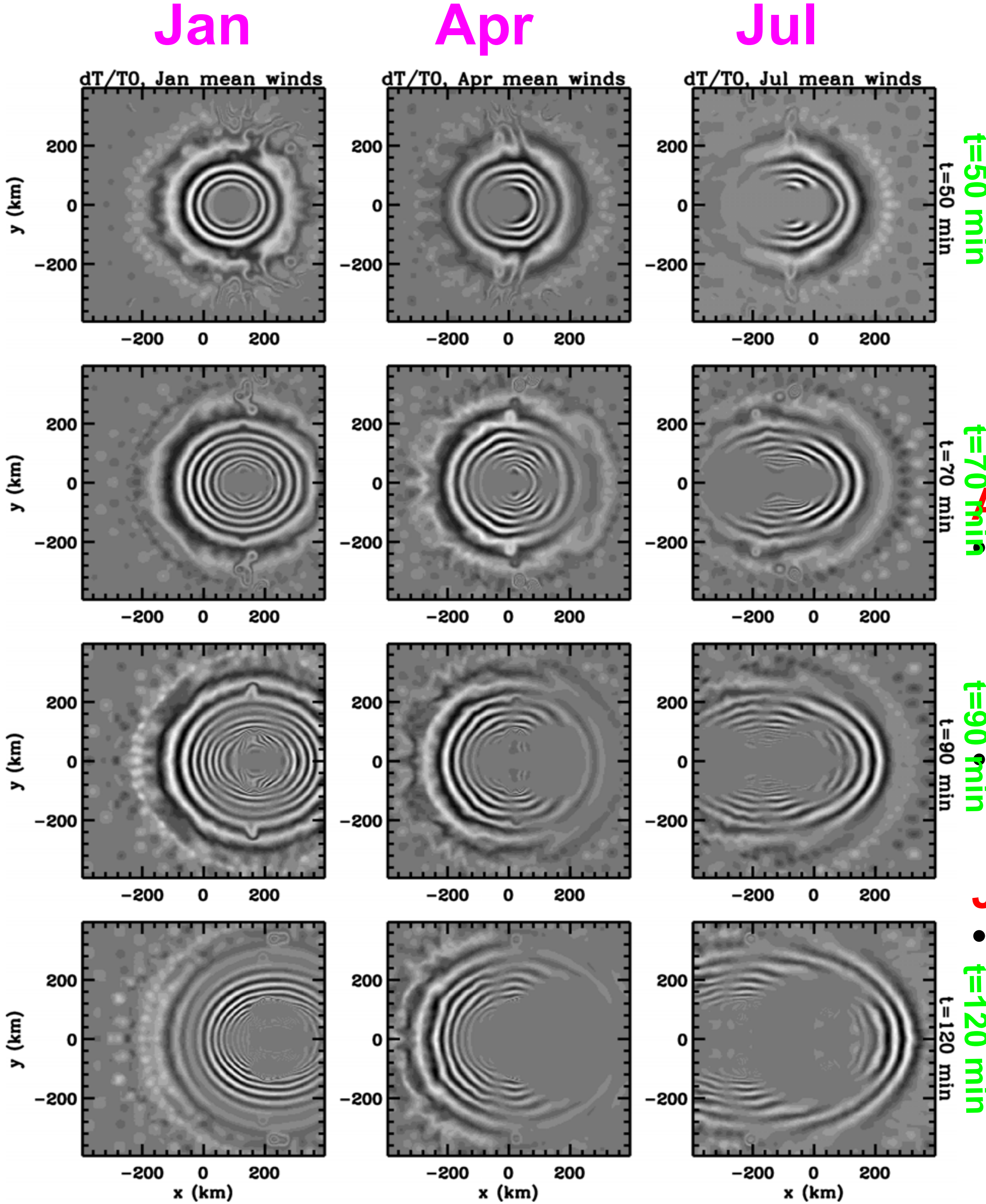
Winds were relatively small

(c)





(Yue et al, 2008, in preparation)



Ray-tracing through HAMMONIA mean zonal winds

assumptions: winds and temperatures are slowly-varying

APRIL mean zonal winds:

- Eastward GWs initially strong, then vanish at later times because of reflection of waves with small horizontal wavelengths (large k)
- At later times, westward waves dominate

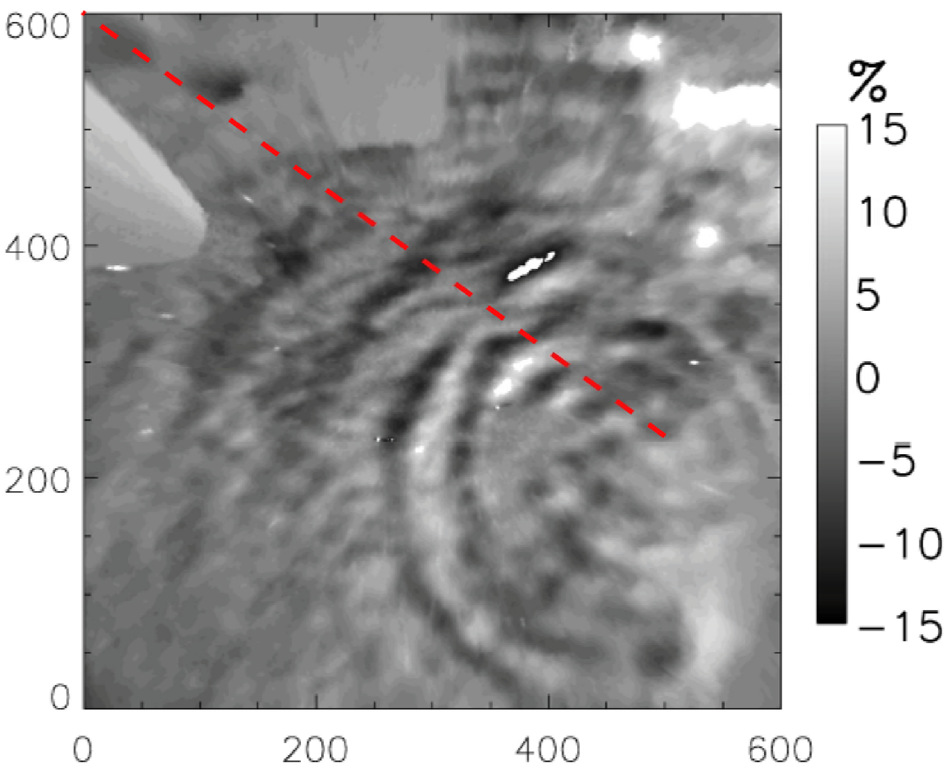
JAN and JUL winds

- Centers of concentric rings shift with time during April and Jul due to strong mean wind filtering on waves with slower phase speeds

(Vadas et al, 2008, in preparation)

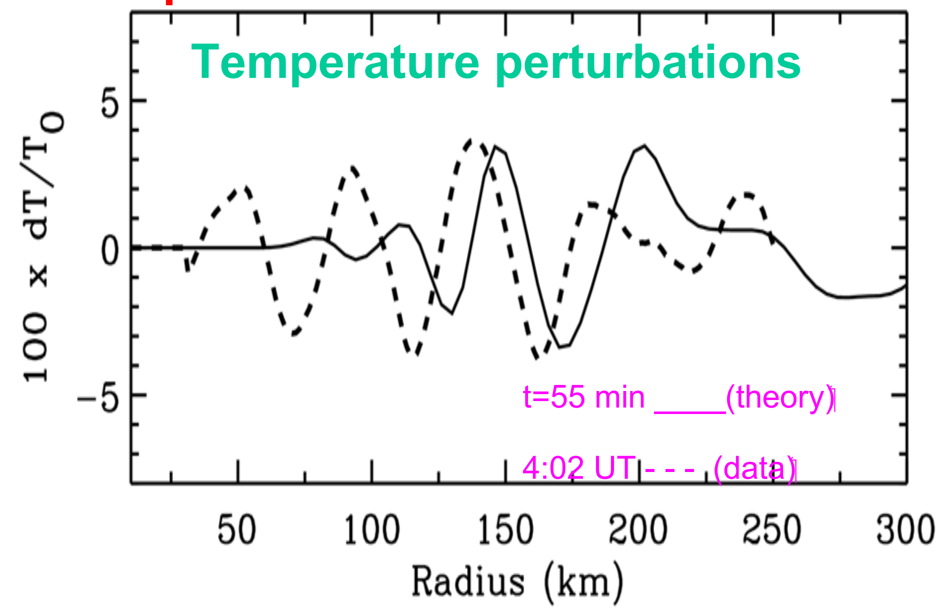
Yucca Ridge OH imager, Colorado 11 May, 2004

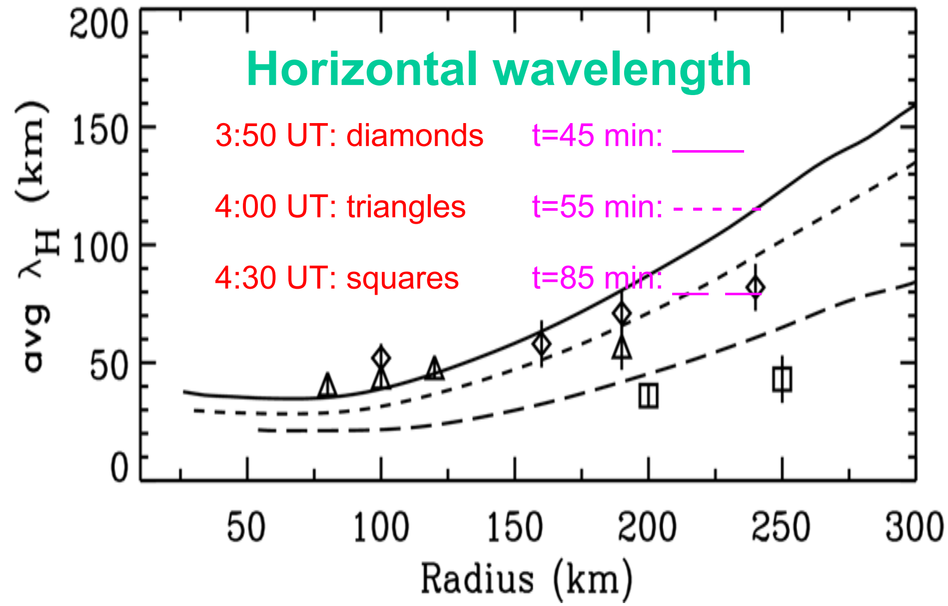
Concentric rings of GWs observed in the OH airglow layer during the equinox



Courtesy of Jia Yue

Ray trace through mean zonal April HAMMONIA winds





(Vadas et al, 2008, in preparation)

Intensities agree well with the convective plume model results

- **4. Determine the neutral response to wave dissipation in the thermosphere from gravity waves from convection**

When GWs dissipate in the thermosphere, they ***accelerate the neutral fluid*** in the direction they were propagating prior to dissipating

MOMENTUM DEPOSITION BY ATMOSPHERIC WAVES, AND ITS EFFECTS ON THERMOSPHERIC CIRCULATION

C. O. HINES

Department of Physics, University of Toronto, Toronto, Canada

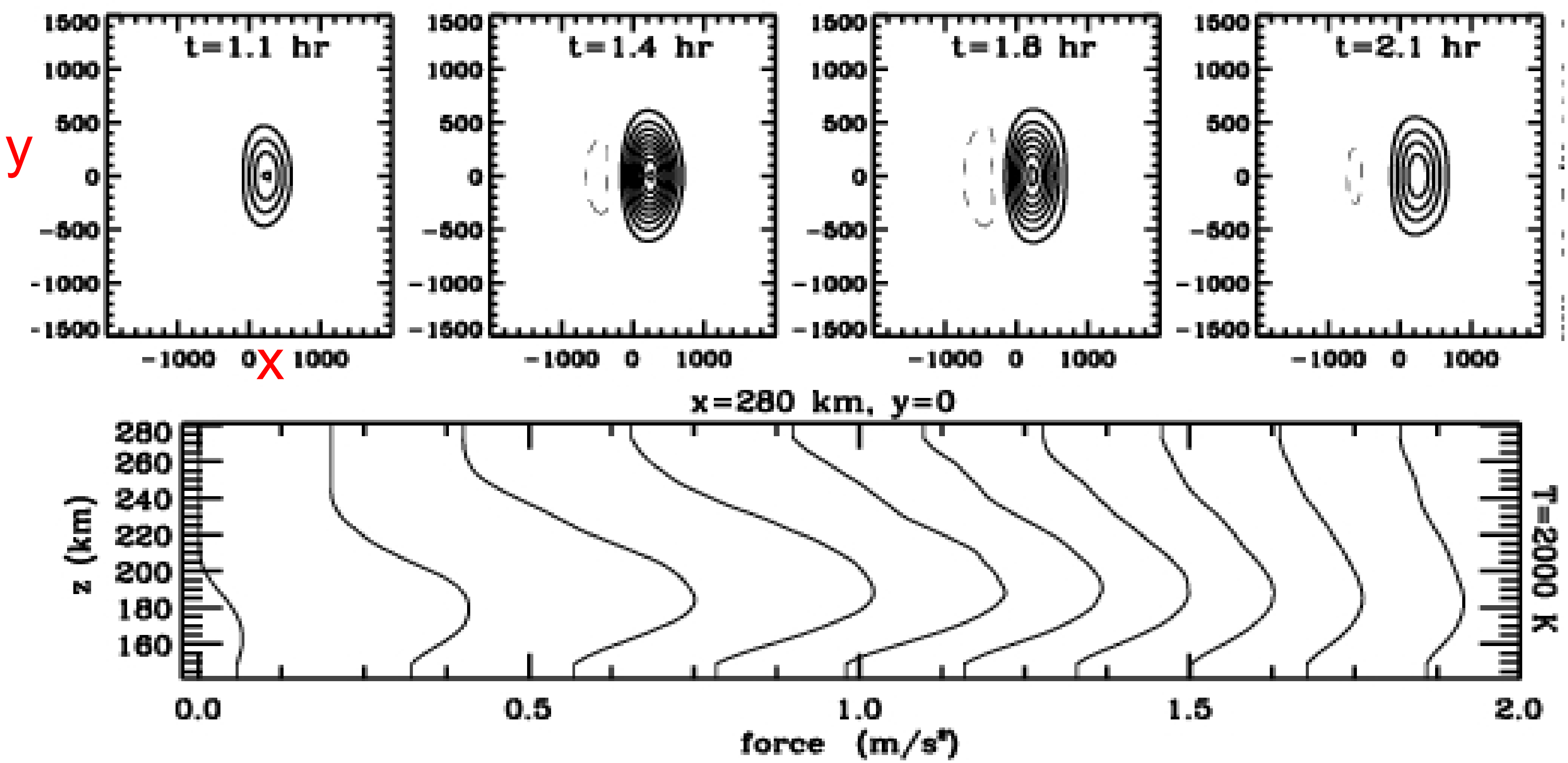
Representative calculations are made for the amount of momentum deposited by atmospheric waves at thermospheric heights, in various circumstances. It is shown that this source of momentum is likely to be a significant one in the circumstances treated. This conclusion may be of importance to an understanding of the winter-time D region at high latitudes, to an understanding of the so-called 'super-rotation' of the upper atmosphere, and to an understanding of the wind fields and ionization anomalies that occur following magnetic substorms.

Atmospheric waves transport momentum. When they dissipate, they transfer that momentum to the background flow of the atmosphere. Applications of

(Hines, Space Res, 1972)

Horizontal thermospheric body forces are generated from dissipating GWs

- Thermospheric body forces are 500-1000 km x 1000-1500 km x 50-100 km deep
- Body forces last for ½ hr for a single convective plume
- Body force amplitudes are strong~ 1-10 m/s^2

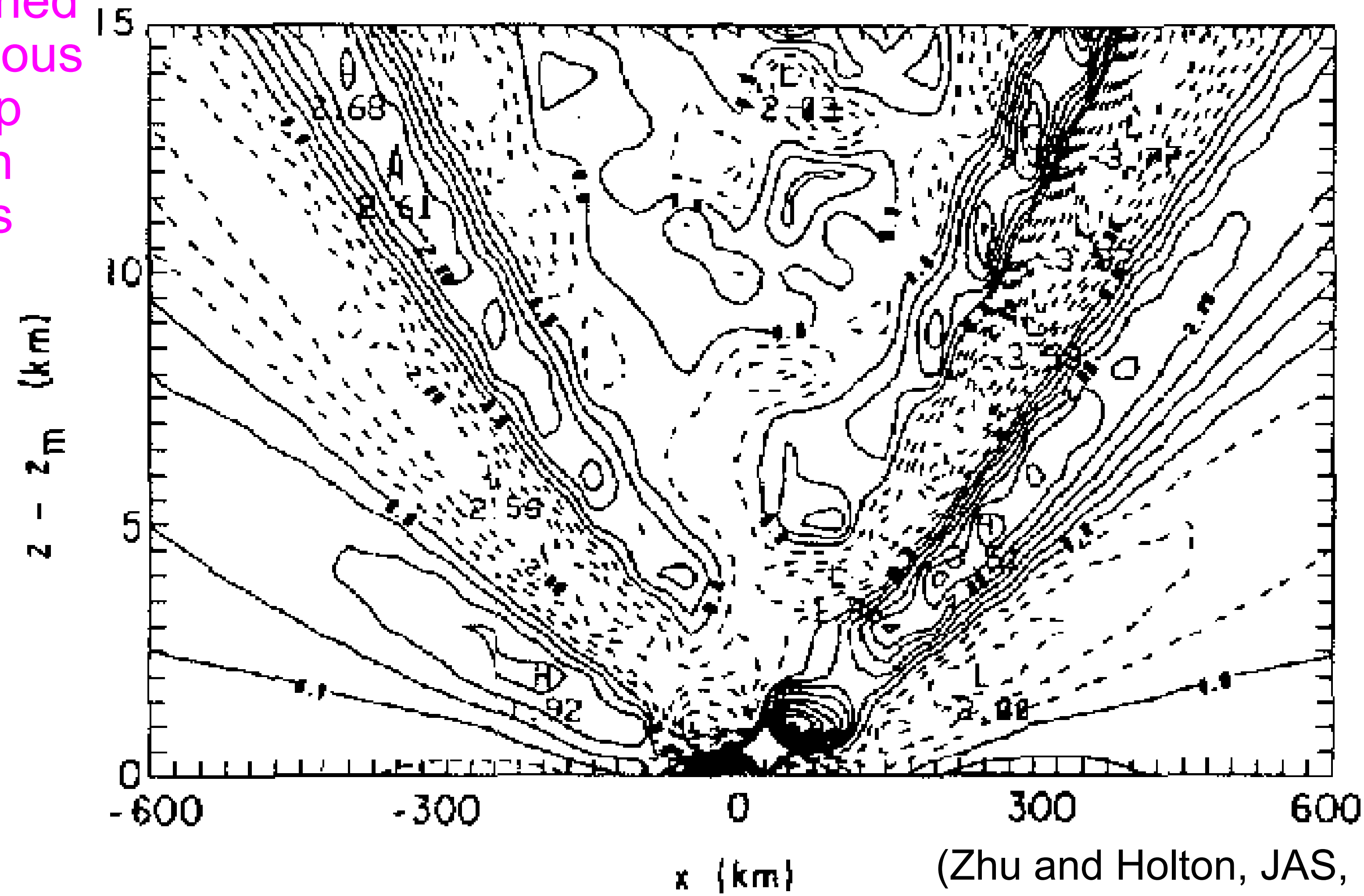


Horizontal slices

(Vadas an Fritts, JGR 2006)

Horizontal body forcings excite GWs

Study examined instantaneous and step function forcings

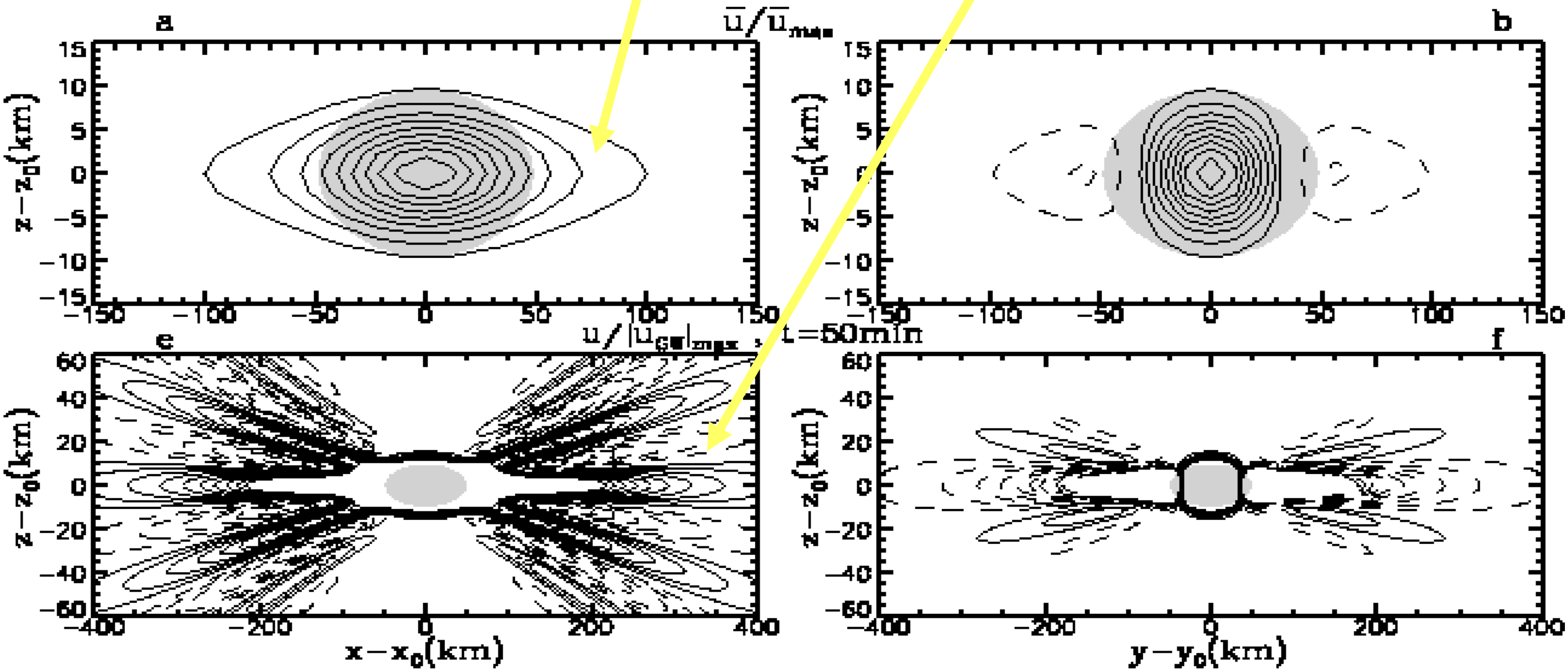


(Zhu and Holton, JAS, 1987)

Mesosphere:

Horizontal body forcings

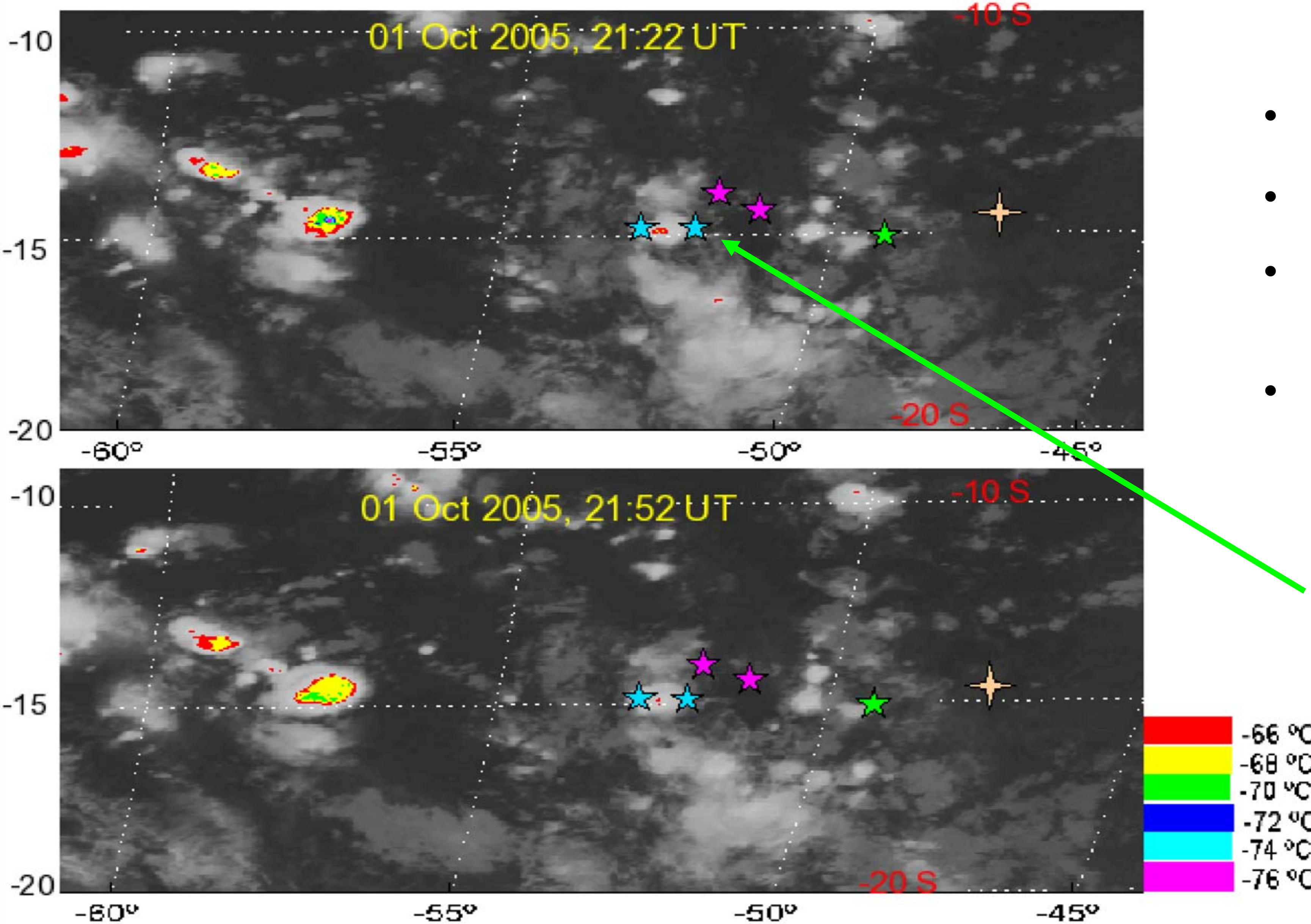
- generate neutral winds
- excite secondary gravity waves



Study examined a horizontal forcing with $\sin^2$ in time

(Vadas etal, JAS, 2003)

Reverse ray-tracing of a medium-scale GW observed in the OH imager above Brasilia, Brazil, during the SpreadFEx campaign



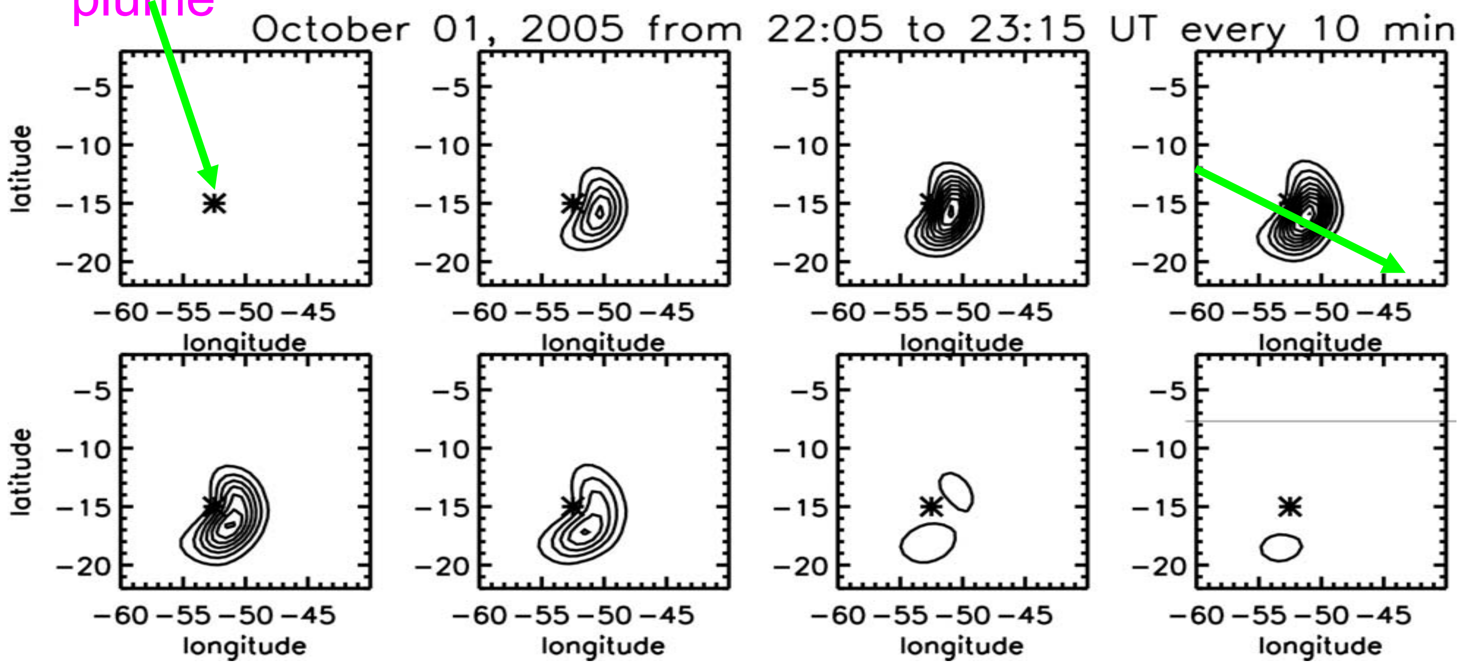
- Oct 1, 23:06 UT
- λ_H~71.4 km
- τ ~ 20.6 min
- Propagating eastward
- Reverse ray-traced to a strong, localized, convective plume with 40 m/s updraft, and 20 km horizontal extent

Vadas etal, 2008, submitted to Annal. Geoph)

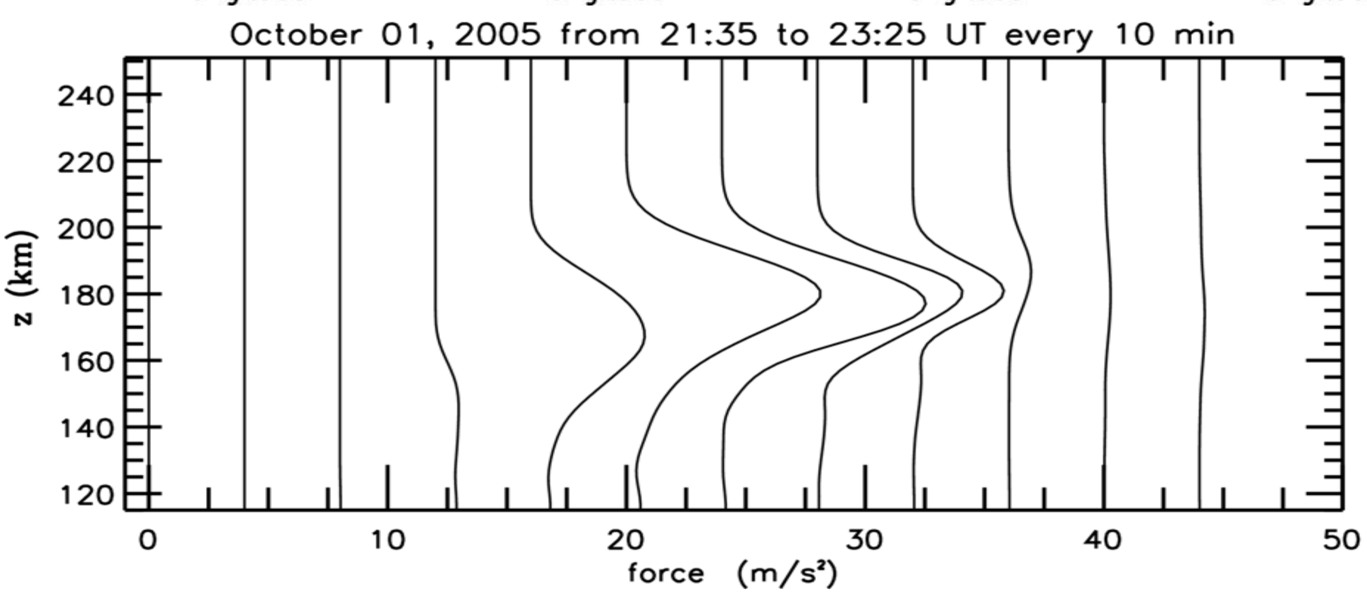
Vertical profile of the created thermospheric body force on 01 Oct, 2005, in Brazil:

These southeastward-propagating GWs created southeastward-propagating body forces where they dissipate in the thermosphere

Convective plume



Body force is southeastward, with a very large amplitude of 9 m/s^2!
Duration is 1.2 hrs, horizontal extent is ~5-8°

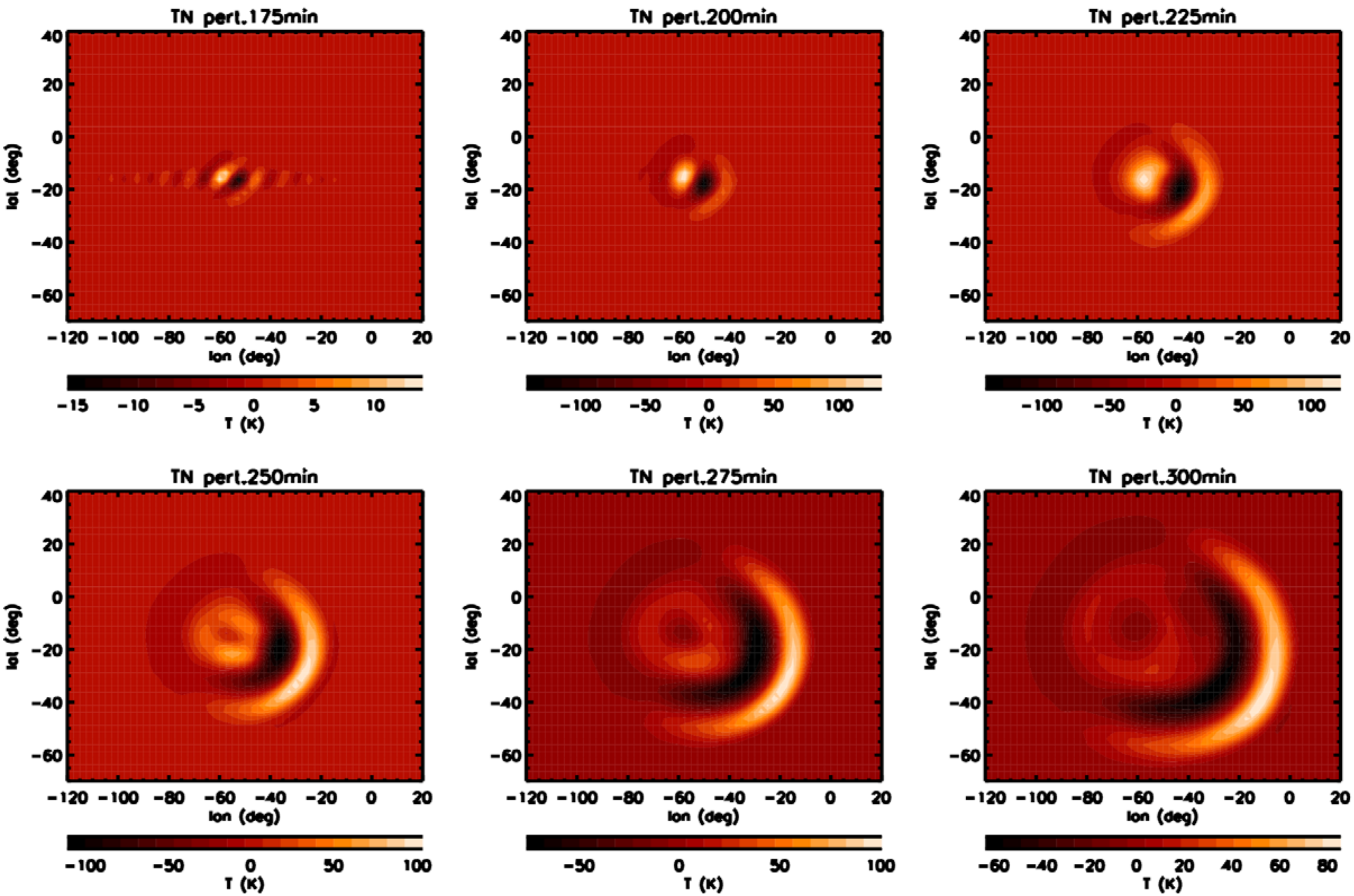


Maximum occurs at z~177 km,
vertical extent is ~50 km

Neutral temperature perturbations:

Southeastward-propagating large-scale secondary GWs are excited by this thermospheric body force.

01 Oct, 2005, in Brazil



- z=250 km,
- times measured from 22:00 UT.

λ_H ~3000 km,

c_H~700 m/s,

τ~1.2 hrs

Courtesy of H-Li Liu

Hedin and Mayr, JGR,1987

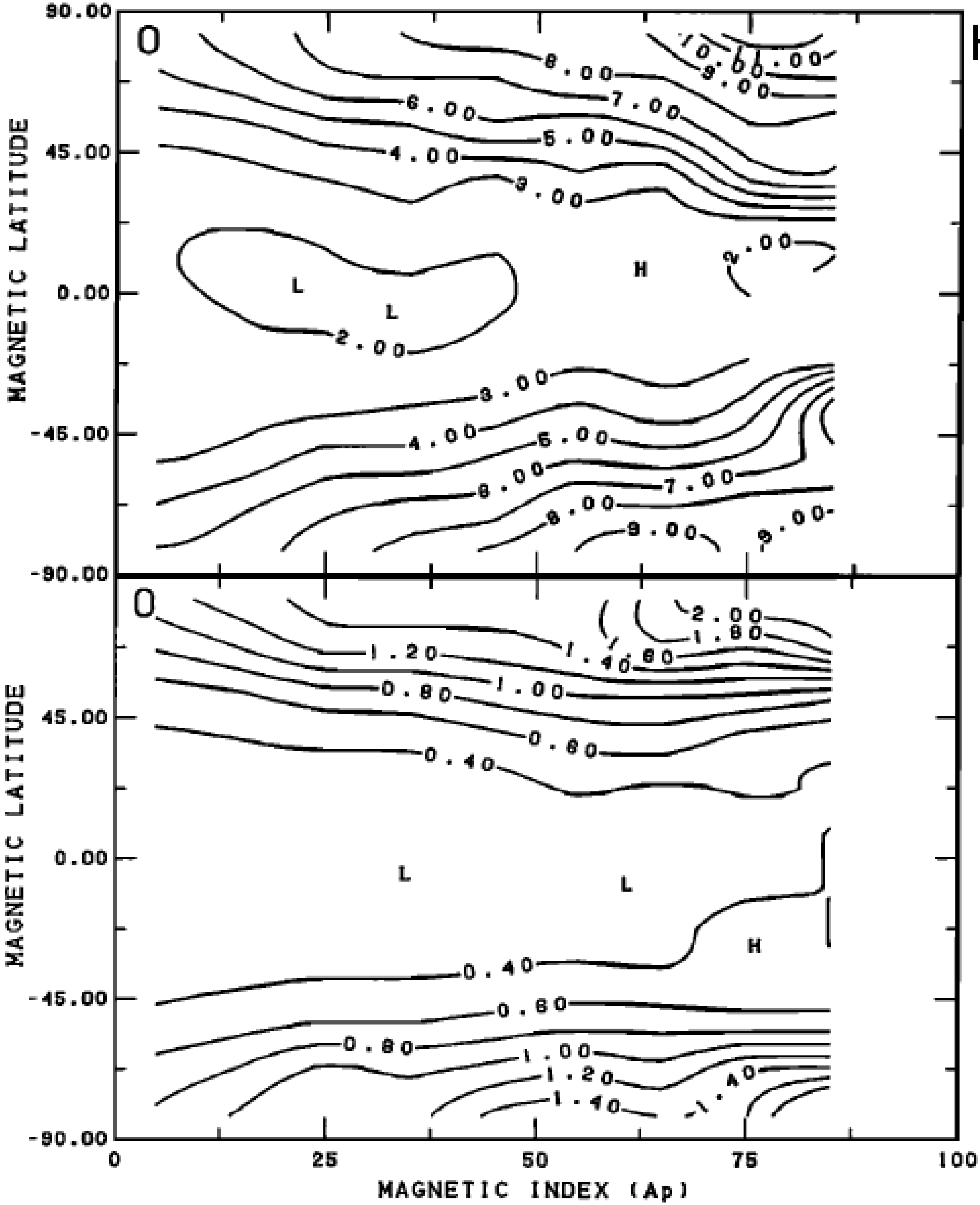


Fig. 7. Contours of rms oxygen density fluctuations (in percent) as a function of magnetic index (Ap) and magnetic latitude for long-wavelength (upper panel) and short-wavelength (lower panel) waves.

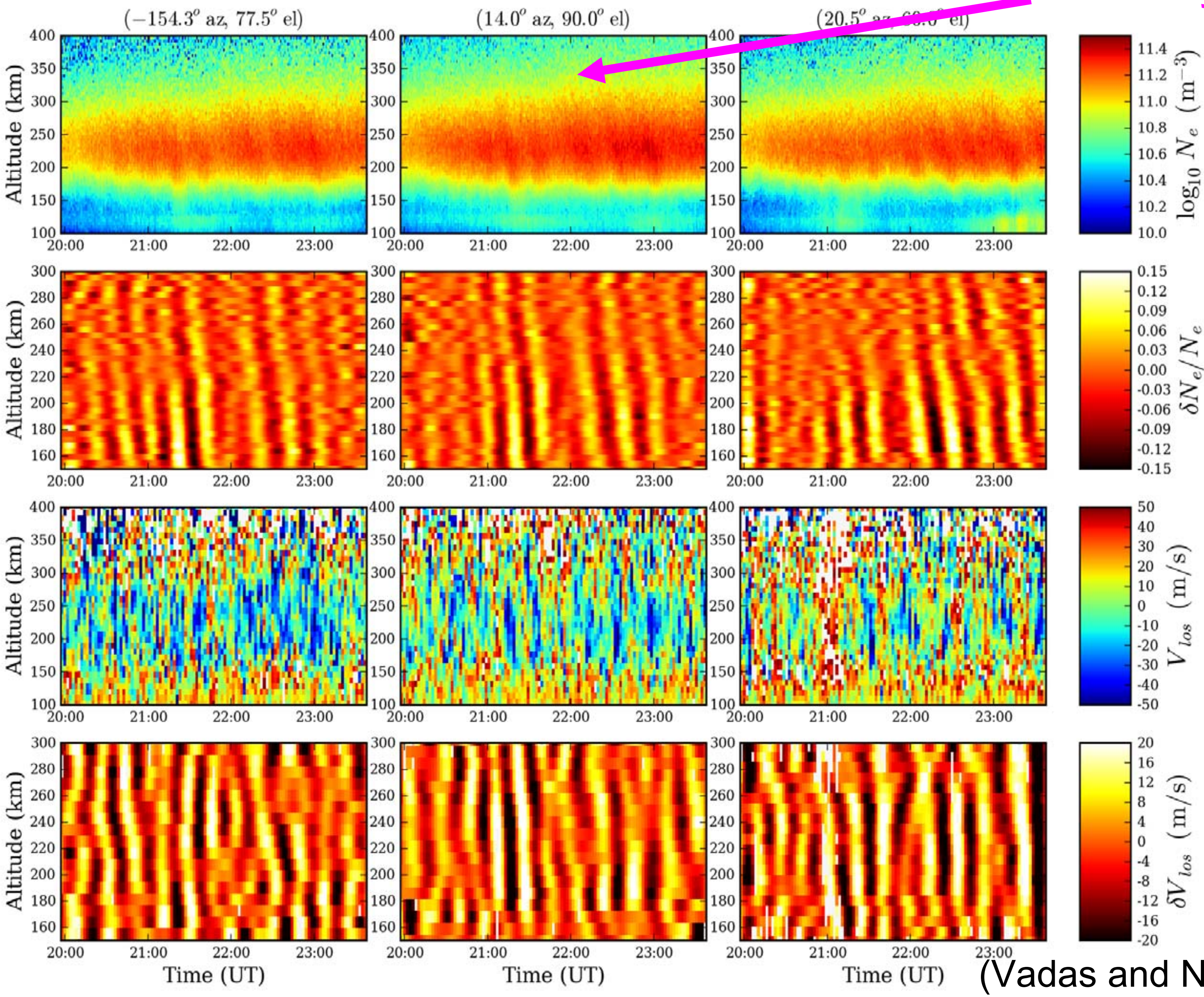
Perturbation amplitudes of short (40-400 km) and long (400-4000 km) wavelengths GWs are virtually independent of the Ap index below 60^o magnetic latitude

z~300 km

- **5. Extract the vertically-varying, neutral horizontal winds from Poker Flat ISR (AMISR) electron density profiles**

Observed Gravity Waves in Thermosphere using AMISR) system in Poker Flat, Alaska (Dec. 13, 2006)

Vertically-pointed beam



electron density

Relative electron density perturbations

Ion velocity perturbations

Band-pass filtered ion velocity perturbations

(Vadas and Nicolls, GRL, 2008)

If one knows λ_{H}, λ_{z}, and the ground-based wave period,

<u>then the wind in the direction of propagation of the gravity wave can be determined iteratively</u>

from the anelastic dissipative, GW dispersion relation.

(this includes the change in *m* from dissipation)

$$\omega_{Ir}^2 = \frac{k_H^2 N^2}{(\mathbf{k}^2 + \frac{1}{4\mathrm{H}^2})(1+\delta_+ + \delta^2/\mathrm{Pr})} \left[1 + \frac{\nu^2}{4\omega_{Ir}^2}\left(\mathbf{k}^2 - \frac{1}{4\mathrm{H}^2}\right)^2 \frac{(1-\mathrm{Pr}^{-1})^2}{(1+\delta_+/2)^2}\right]^{-1}$$

δ and δ_+ depend on *m* and ω_{Ir}

Solve for ω_{Ir}, then U_H

$$\omega_{Ir} = \omega_r - kU - lV = \omega_r - k_H U_H$$

Intrinsic frequency

observed frequency

Zonal wind U

Meridional wind V

Wind along direction of propagation

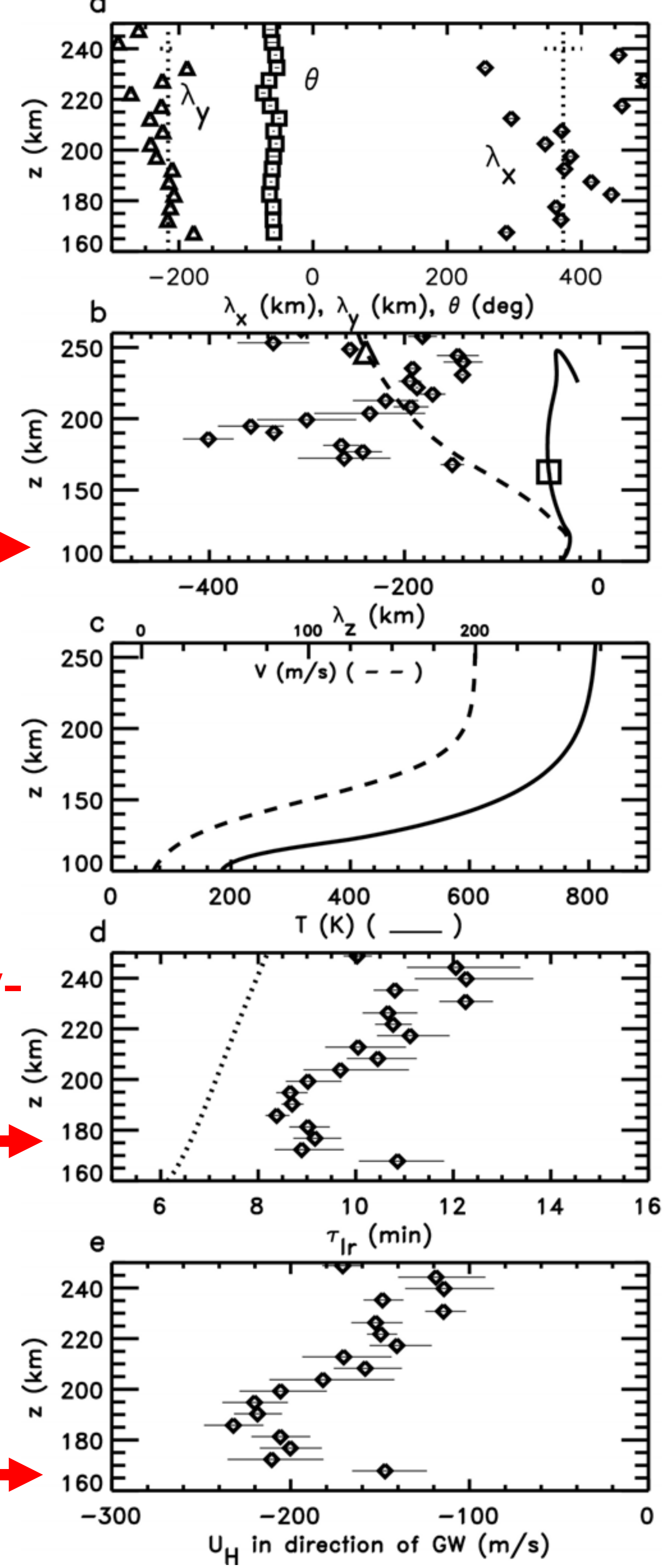
Vertically-pointed beam

- Conditions fairly quiet geomagnetically (ion velocities only 10-20 m/s)
- assume a single ion species of O^+ (a good approximation above 200 km)
- Assume that ion drag causes v_{ion}=w, since magnetic field is nearly vertically-pointed at PFISR
- electron density continuity equation shows that the wave phase is same for GW and electron density perturbation
- λ_z can be computed by taking a vertical derivative of the relative electron density perturbation along the "lines" of constant phases

Calculated vertically-varying λ_z from data

Computed vertically-varying intrinsic period

Wind along propagation direction

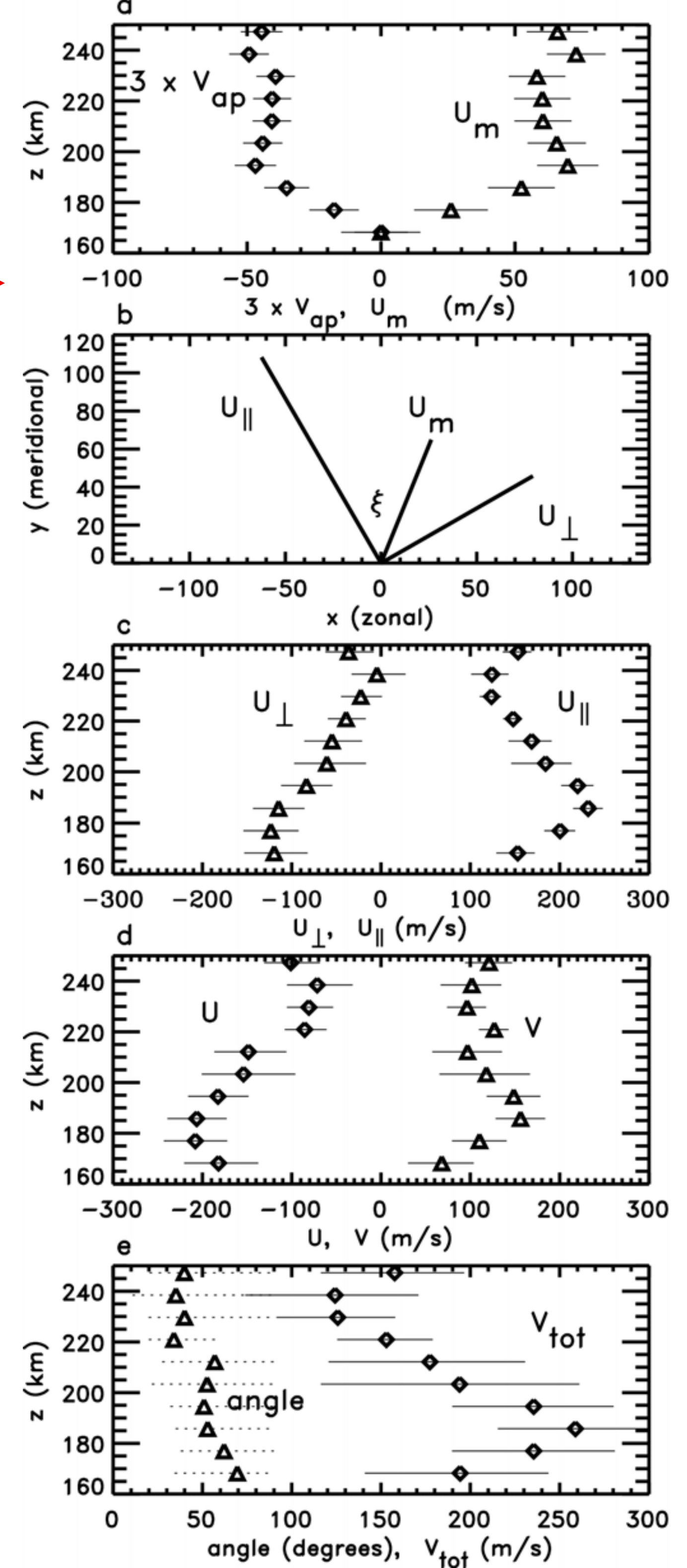


(Vadas and Nicolls, GRL, 2008)

V_{ap} is mean component of the anti-parallel ion velocity, U_m is neutral wind along magnetic meridian (neglecting diffusion, which may be important)

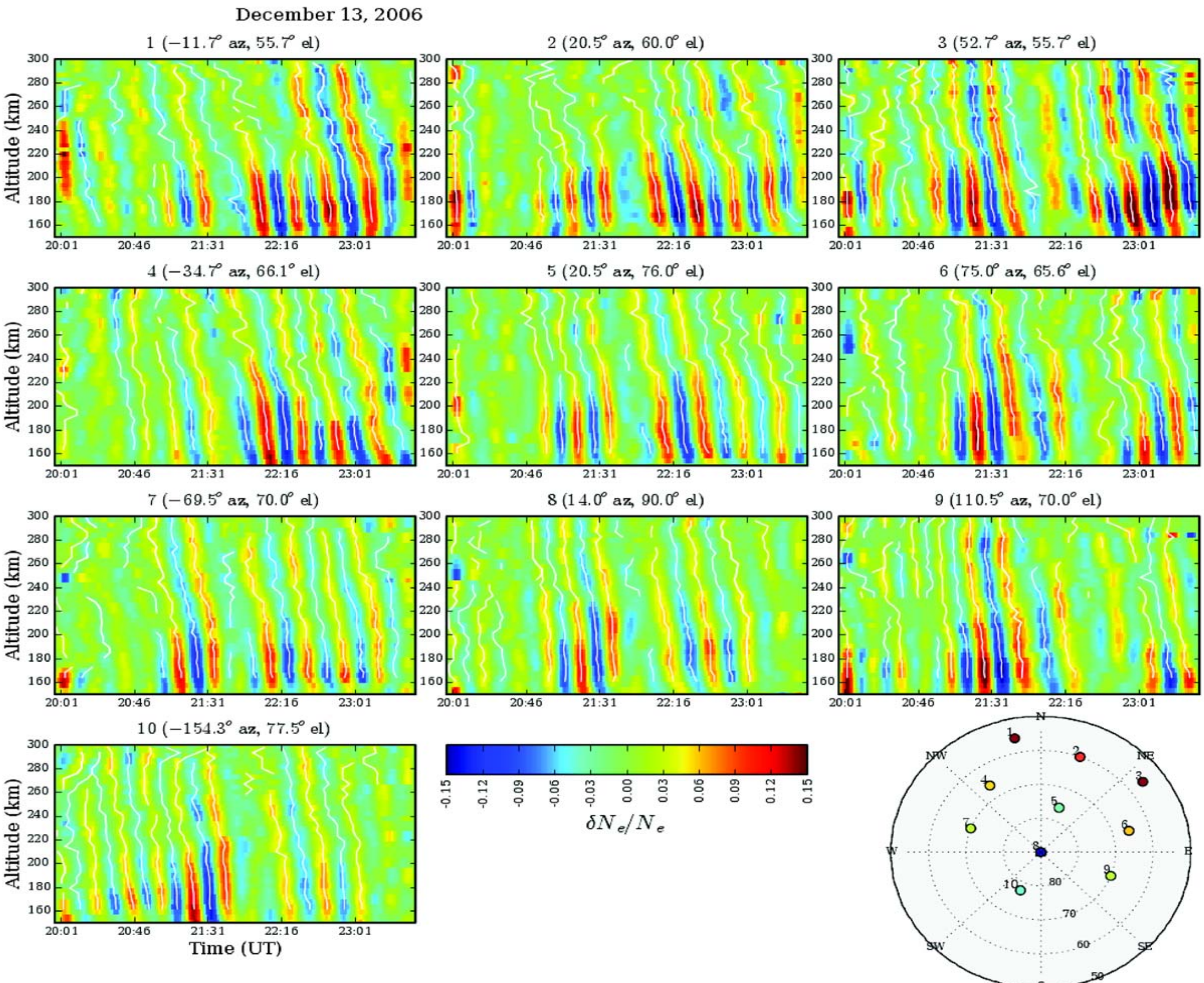
Can extract the total neutral, background wind from 180-250 km using this dissipative, anelastic dispersion relation if can calculate neutral wind along magnetic meridian

Using simple geometry, calculated vertically-varying zonal and meridional winds over PFISR



(Vadas and Nicolls, GRL, 2008)

Observed Gravity Waves in all 10 beams using PFISR

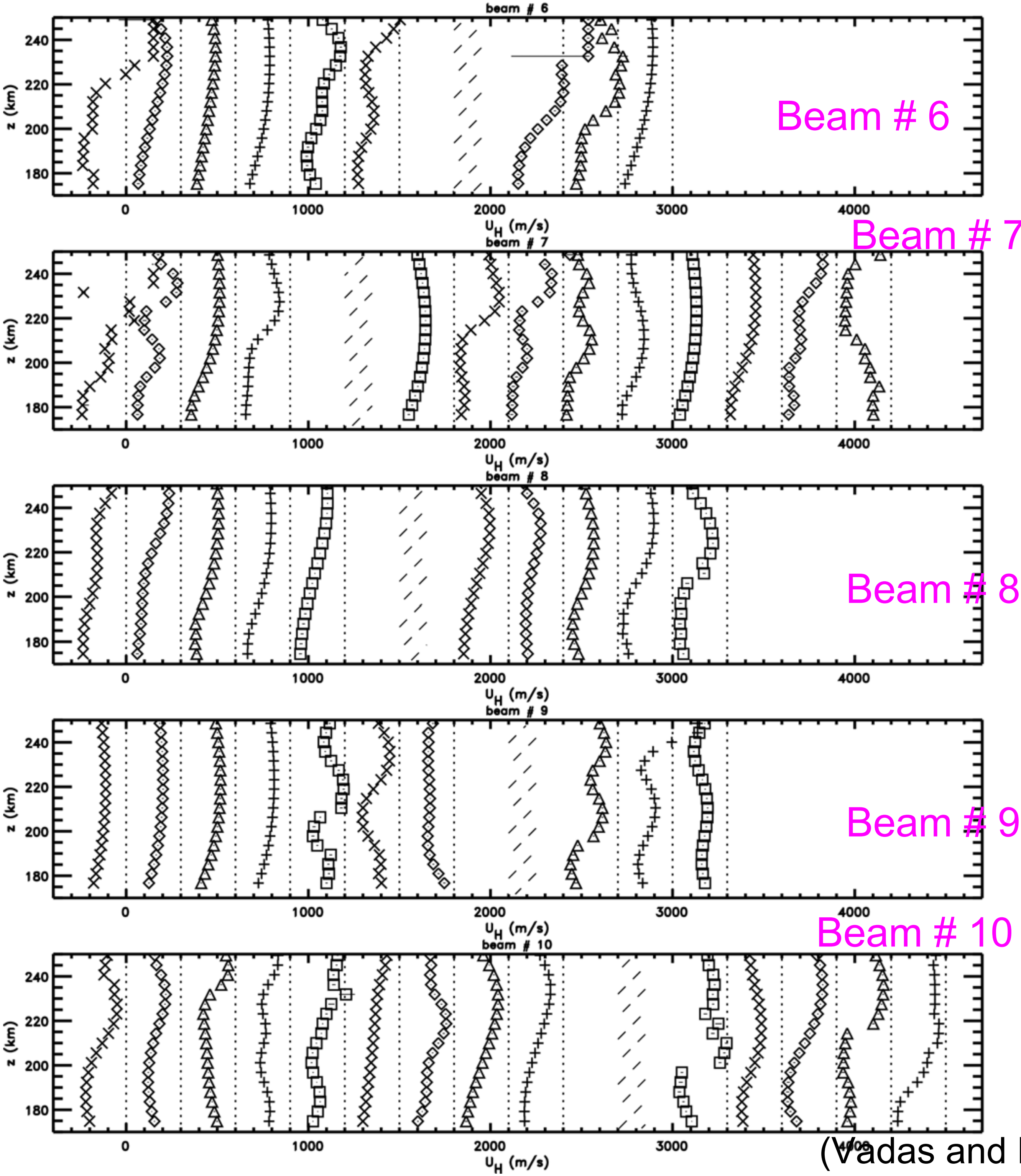


Relative electron density perturbations

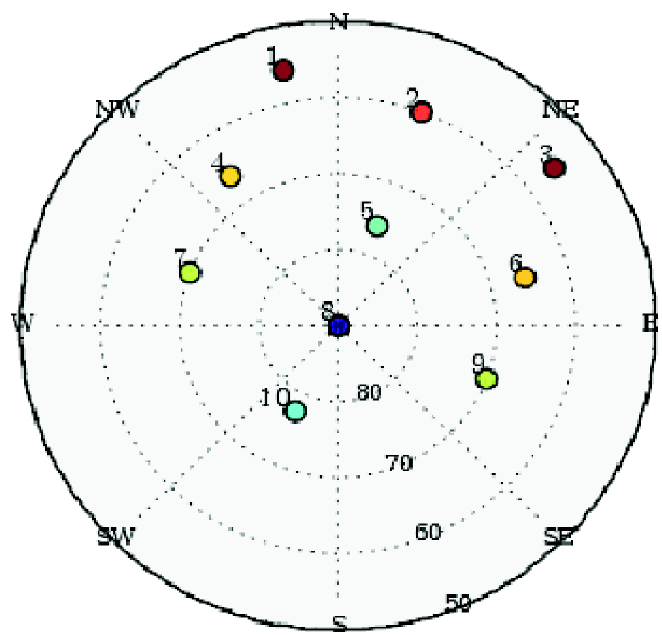
- GW_1: λ_H=180 km, τ_r=20 min, propagating SEward
- GW_2: λ_H=200 km, τ_r=24 min, propagating SEward

(Vadas and Nicolls, 2008, submitted to JASTP)

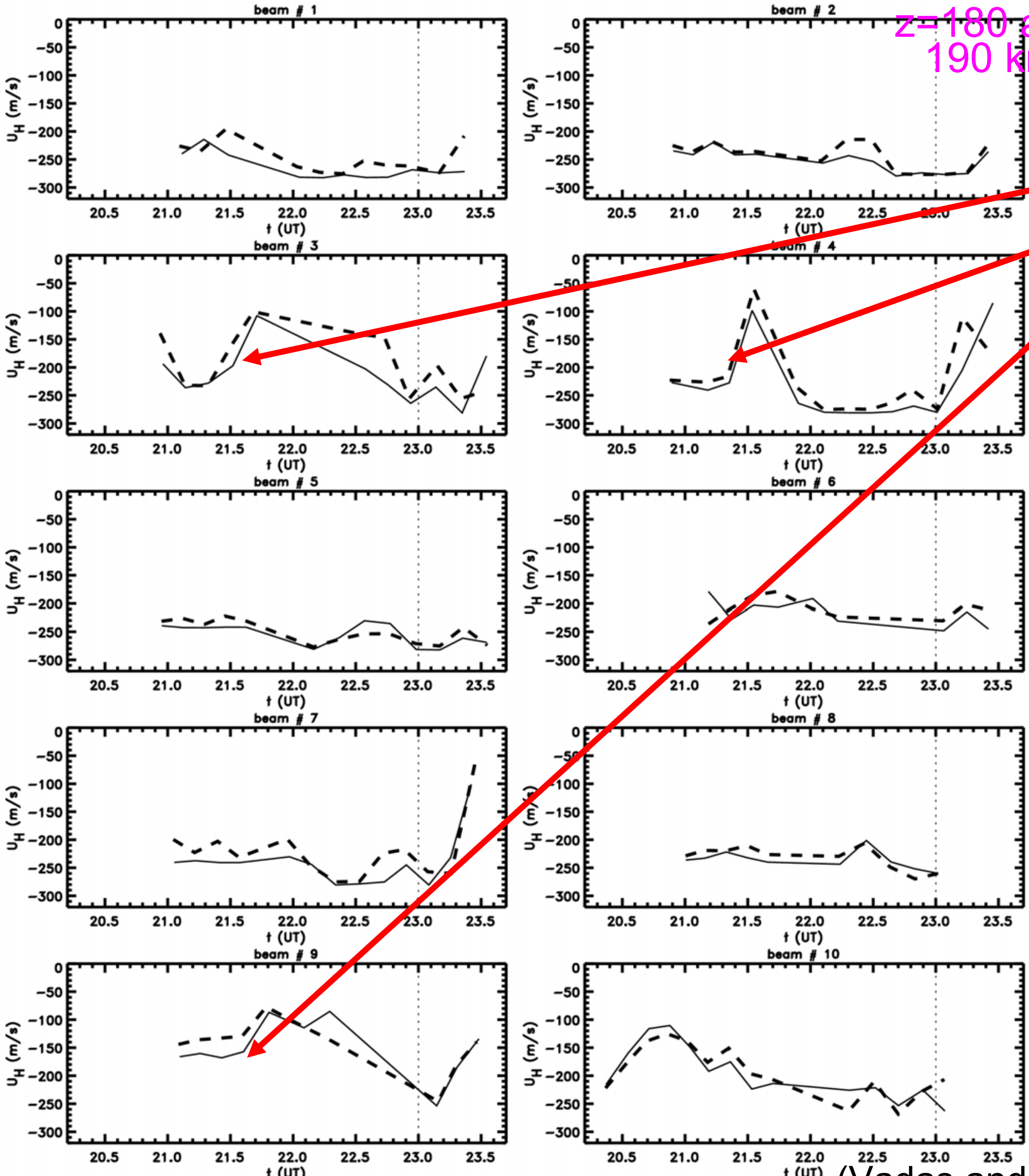
Extract the neutral, background wind profile for each constant wave phase line.
Then know how the winds evolve in time.



Extracted a
- 4.5-6 hr large-scale wave with λ_z=80 km
- A third medium-scale wave with a 22 min period propagating SEward

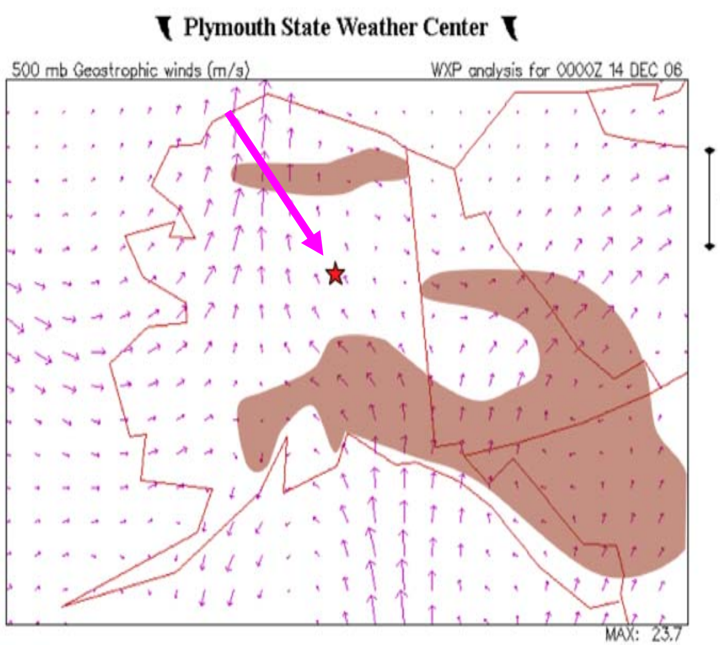


(Vadas and Nicolls, 2008, submitted to JASTP)



SEward thermospheric accelerations at the same times in nearby beams of 0.1-0.2 m/s^2

These accelerations are likely caused by the dissipation of gravity waves from a source NW of Poker Flat.



(Vadas and Nicolls, 2008, submitted to JASTP)

Conclusions

The dissipative anelastic gravity wave dispersion relation is useful for

- Understanding the effects of dissipative filtering on wave scales and altitudes
- Ray-trace studies coupling the lower atmosphere with the mesosphere and thermosphere
- Exploring the role convection plays in the thermospheric dynamics
- Extracting the vertically-varying neutral thermospheric wind profiles (and inferring neutral dynamics) from PFISR electron density profiles