A Novel Data Assimilation Model for the Plasmasphere

CEDAR Workshop 2017

AGS PostDoc Report

AGS-PRF: AGS-1231207

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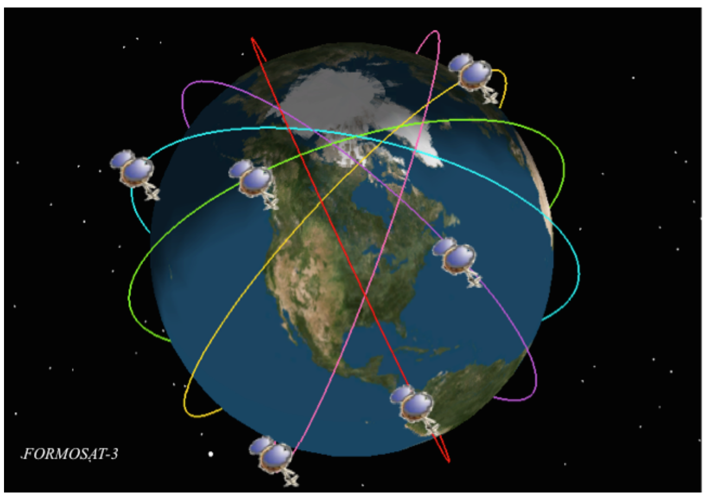
Objective and Motivation

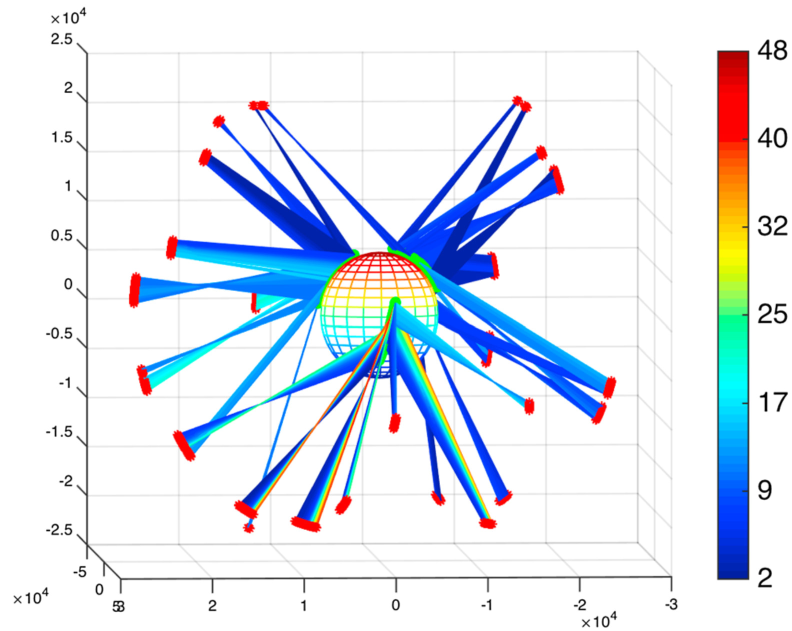
- **Objective:**
  - **Apply the idea of data assimilation to the plasmasphere**
  - **To develop a plasmaspheric data assimilation technique to produce time-evolving global maps of plasmasphere electron density structures.**

- **Motivation**
  - **Understanding magnetosphere/ionosphere coupling requires understanding the dynamics of the transition region between the two, Earth's plasmasphere.**
  - **Plasmaspheric dynamics is poorly characterized due to the paucity of in-situ data available, making it difficult to construct a global plasmasphere picture.**
  - **Earlier tomographic imaging efforts were necessarily limited to 2D electron density slice within the LEO orbit plane.**
  - **Data assimilation is a mathematical framework for the statistical union between observations and empirical or physics-based models.**

Methodology – Data Assimilation

- Based on Ionosphere Data Assimilation 4D (IDA4D) [Bust et al. 2004]
  - **Data:**
    - COSMIC/FORMOSAT-3: Total electron content (TEC)
    - Precise Orbit Determination
      - Overhead GPS TEC signals





Space APL

Methodology – Data Assimilation

- **Forward model:**
  - **Relating observations to state variables (electron density)**
    - **Line-of-sight integration**

$$TEC = \int N_e(z)dz + \varepsilon_k \quad \text{or} \quad y_k = H_k x_k + \varepsilon_k$$

- **Inverse model:**
  - **Statistical framework:**
  - $\varepsilon_k \sim \mathcal{N}(0,\ R_k)$, $p(y_k|x_k) \sim \exp\left(-\frac{1}{2}\left(\|y_k - H_k x_k\|^2_{R_k^{-1}}\right)\right)$
  - **Maximum-Likelihood (ML) estimate :** $x_k^{ML} = max\, p(y_k|x_k)$

$$x_k^{ML} = \underset{x_k}{\arg\min}\ \frac{1}{2}\left(\|y_k - H_k x_k\|^2_{R_k^{-1}}\right)$$

- H is ill-conditioned, in general. Cannot solve directly via $x_k = H_k^{-1} y_k$

Methodology – 3D-VAR

1) Analysis: Compute the estimate at each time step

- **Bayesian Approach *(a priori* information*)*:**
  - **Model:**
    - Global Core Plasma (GCP) Model [Gallagher et al., 2000]
    - Empirical, Kp-driven

  - $\boldsymbol{x_k \sim \mathcal{N}\left(x_k^b,\ P_k^b\right)}$
  - **Maximum *a posteriori* Estimate:** $\boldsymbol{x_k^a} = \underset{x_k}{\arg\max}\, \boldsymbol{p(y_k|x_k)p(x_k)}$
    - Objective function combining the deviation from the data and the model
    - 3D-VAR: estimate 3D structure of the unknown state

$$\boldsymbol{x_k^a} = \underset{x_k}{\arg\min}\, \frac{1}{2}\left(\|\boldsymbol{y_k - H_k x_k}\|^2_{R_k^{-1}} + \left\|\boldsymbol{x_k - x_k^b}\right\|^2_{P_b^{-1}}\right)$$

    - Analytical expressions for the solution exist:

$$\boldsymbol{x_k^a = x_k^b + P_k^b H_k^T\left[R_k + H_k P_k^b H_k^T\right]^{-1}\left(y_k - H_k x_k^b\right)}$$

$$\boldsymbol{P_k^a = P_k^b - P_k^a H_k^T\left[R_k + H_k P_k^b H_k^T\right]^{-1} H_k P_k^b}$$

Methodology – Plasmasphere Data Assimilation (PDA)

- **We find that with the ingestion of upward looking data alone, the problem remains ill-posed.**
- **Hence, we further constrain the solution to be vertically smooth.**
- $\boldsymbol{x_k \sim \mathcal{N}\left(x_k^b,\ [P_k^{-1} + \lambda^2 D^T D]^{-1}\right)}$

$$\boldsymbol{x_k^{aMAP}} = \arg\min_{x_k} \frac{1}{2}\left(\|\boldsymbol{y_k - H_k x_k}\|^2_{\boldsymbol{R_k^{-1}}} + \left\|\boldsymbol{x_k - x_k^b}\right\|^2_{\boldsymbol{P_b^{-1}}} + \boldsymbol{\lambda^2}\left\|\boldsymbol{D(x_k - x_k^b)}\right\|_{\boldsymbol{I}}^2\right)$$

Where D and λ are the regularization functional and regularization parameter, respectively.

- **How to solve this analytically?**

$$\boldsymbol{x_k^{aMAP}} = \arg\min_{x_k} \frac{1}{2}\left(\boldsymbol{\lambda^2}\left\|\boldsymbol{D(x_k - x_k^b)}\right\|^2 + \|\boldsymbol{x_k - x_k^a}\|^2_{\boldsymbol{P_a^{-1}}} + \boldsymbol{C}\right)$$

- **3-D VAR:** $$\boldsymbol{x_k^a} = \arg\min_{x_k} \frac{1}{2}\left(\|\boldsymbol{y_k - H_k x_k}\|^2_{\boldsymbol{R_k^{-1}}} + \left\|\boldsymbol{x_k - x_k^b}\right\|^2_{\boldsymbol{P_b^{-1}}}\right)$$

- **PDA Two-stage analysis:**

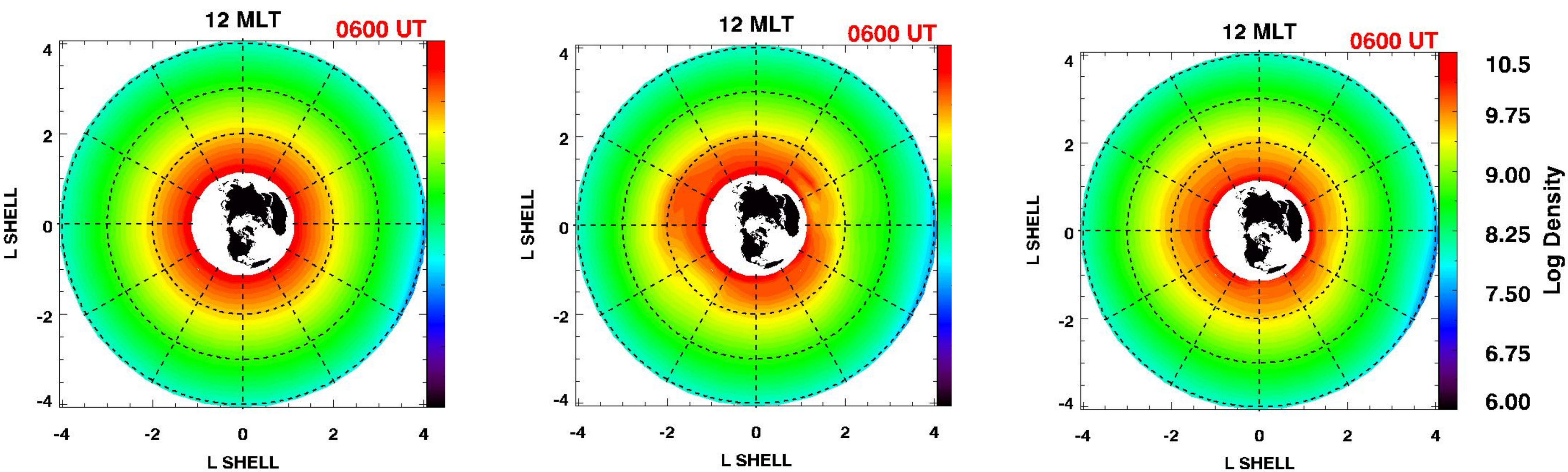
i. $\boldsymbol{x_k^a = x_k^b + P_k^b H_k^T [R_k + H_k P_k^b H_k^T]^{-1} (y_k - H_k x_k^b)}$

ii. $\boldsymbol{x_k^{aMAP} = x_k^a + P_k^a D^T \left[I + D P_k^a D^T\right]^{-1} (x_k^b - D x_k^a)}$

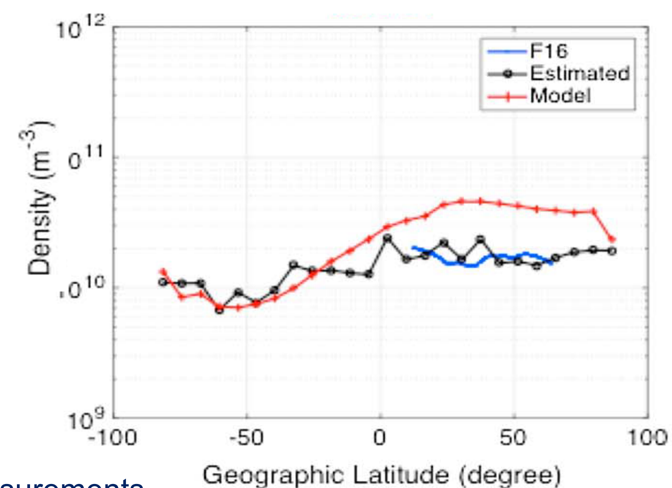
2) Prediction: Moving the state forward in time via Gauss-Markov Kalman.

PDA Estimation results – 1 Quiet Day, 06/19/2009

Model | 3D-VAR | PDA



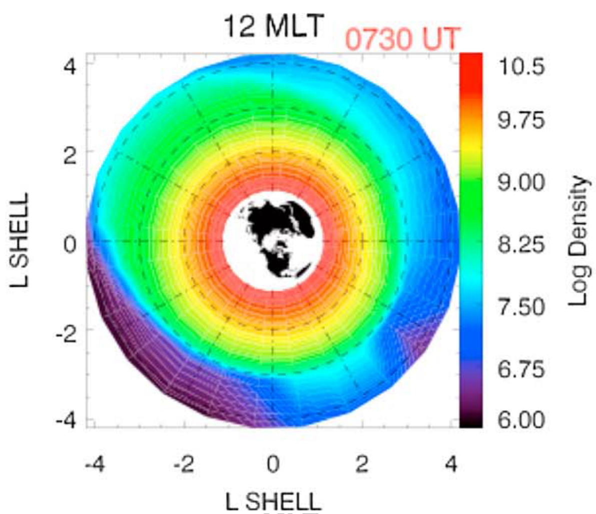
- Equatorial cross sections of the plasmasphere electron density (m^{-3}).
- The cross sections extend to L=4 (20,000 km).
- Sun is at the top of the image, dawn to the right.
- PDA eliminates unrealistic altitude gradients.
- A good agreement is demonstrated between DMSP-F16 in-situ densities and PDA estimated densities.



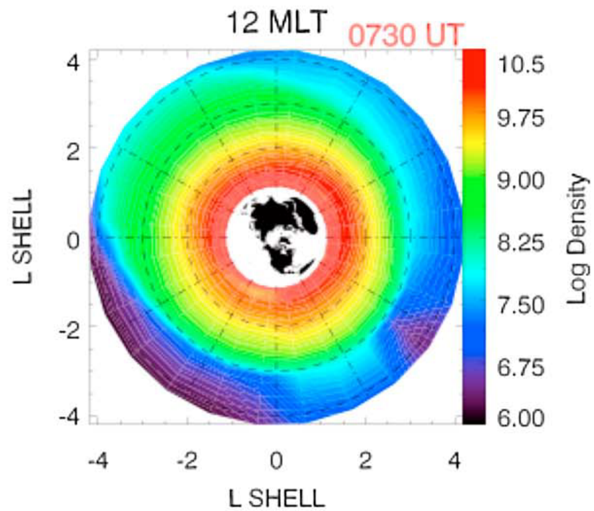
The authors are grateful to Dr. Hairston from UT Dallas for providing DMSP density measurements

PDA Estimation results – 2 Storm Day, 03/17/2013

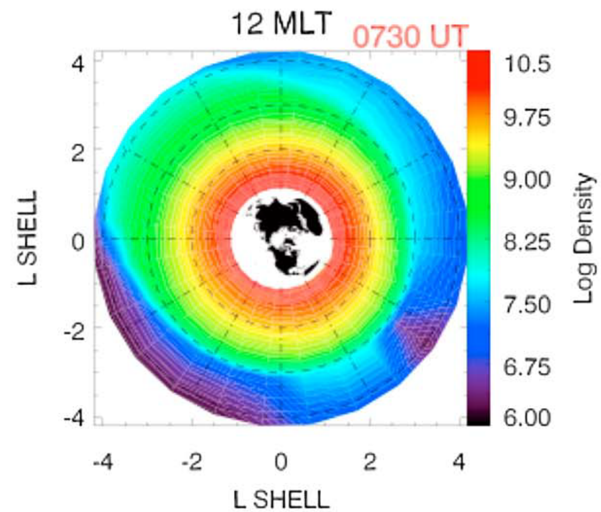
Model

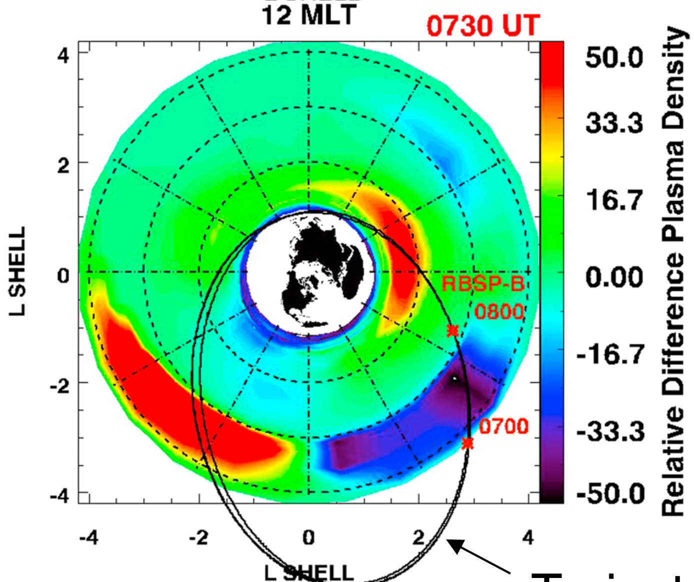


3D-VAR



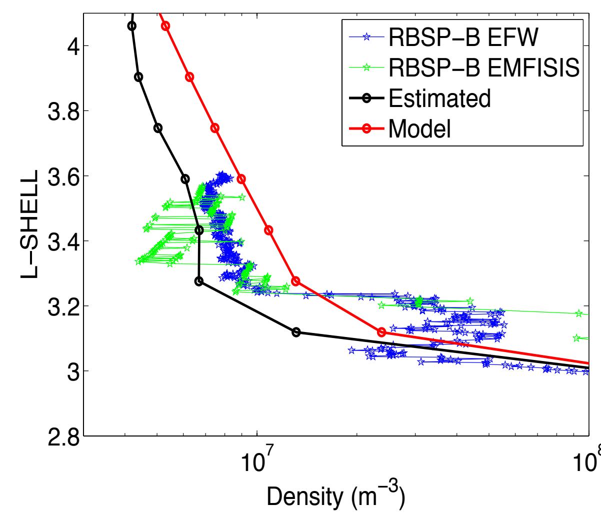
PDA





Absolute difference between the background model and the PDA estimate weighted by model density on 17 March 2013, 0730 UT.

Trajectory of the RBSP-B



The authors are grateful Wygant and Thaller from the University of Minnesota, and to Kurth and De Pascuale from the University of Iowa

Summary and Future Directions

- **An assimilative model that provides global 3D maps of electron density in the plasmasphere.**
- **For plasmasphere data assimilation, it is necessary to impose a smoothness constraint to avoid non-physical gradients.**
- **A coupled ionosphere-plasmasphere data assimilation model****
  - **The extension of the PDA grid to lower ionospheric heights**
    - **Ground-based GPS TEC**
    - **ISR measurements**
    - **FUV measurements**
    - **COSMIC-2 GPS TEC**
  - **Inclusion of physics-based model such as RCM-E and SAMI**
  - **3D Evolution of the system during geomagnetic storms (plumes, SED [Foster et al, 2002])**
- **An integrated ionosphere-plasmasphere assimilative model for more accurate specification, nowcast, and forecast of the upper atmosphere**

[*] NASA-HSR grant #NNX16AG65G, R. Bishop, A. Coster, G. Bust, R. Nikoukar, D. Turner, and C. Lemon, Storm-time Dynamics of the Plasmapause and the Ionosphere/Magnetosphere System, 2016-2018.

[**] G. Bust, A. Chartier, R. Schaefer, R. Nikoukar, E. Miller, IDA2017: A Next-Generation Coupled Modular Assimilation Package, JHU/APL.

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